QUANTIFICAÇÃO DE INCERTEZAS NAS FREQUÊNCIAS NATURAIS DE PLACAS COMPOSTAS: UMA ABORDAGEM DE SIMULAÇÃO DE MONTE CARLO

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RESUMO: *Este estudo procura quantificar os efeitos da incerteza nas propriedades dos materiais sobre as características de vibração das estruturas, com foco nos setores em que a precisão e a confiabilidade são cruciais, como o aeroespacial. As frequências naturais e as respostas dinâmicas podem ser drasticamente alteradas por variações em características como densidade e módulo de Young, que podem ser causadas por variações nos processos de fabricação, fatores ambientais e deterioração do material. Devido a essas modificações, o comportamento vibracional se torna mais variável, tornando a modelagem probabilística necessária para fazer previsões precisas. Os modos de vibração são ainda mais complicados devido às incertezas nos materiais compostos, como o ângulo das camadas, que alteram a distribuição de massa e a rigidez estrutural. Este estudo usa análise de sensibilidade global e simulações de Monte Carlo, juntamente com modelos de elementos finitos, para avaliar os impactos das variações das propriedades dos materiais na dinâmica estrutural, a fim de resolver esses problemas. Ao criar modelos probabilísticos confiáveis que levam em conta essas incertezas, o objetivo final é melhorar a segurança e a confiabilidade das estruturas. Isso melhorará o projeto e a análise de estruturas compostas que podem suportar fatores ambientais e demandas operacionais. Este trabalho parte do pressuposto de que esses modelos podem prever com precisão o comportamento dinâmico, resultando em projetos estruturais mais seguros e eficazes.*

PALAVRAS-CHAVE: *Simulação de Monte Carlo, Frequências Naturais, Análise de Sensibilidade, Quantificação de Incertezas.*

QUANTIFICATION OF UNCERTAINTIES IN NATURAL FREQUENCIES OF COMPOSITE PLATES: A MONTE CARLO SIMULATION APPROACH

ABSTRACT: *This study attempts to quantify the effects of uncertainty in material properties on the vibration characteristics of structures, with a focus on industries where precision and dependability are crucial, such as aerospace. Natural frequencies and dynamic responses can be dramatically changed by variations in characteristics like density and Young's modulus, which can be caused by variations in manufacturing processes, environmental factors, and material deterioration. Because of these modifications, vibrational behavior becomes more variable, making probabilistic modeling necessary to make precise predictions. Vibration modes are further complicated by uncertainties in composite materials, such as ply angle, which alter mass distribution and structural stiffness. This study uses global sensitivity analysis and Monte Carlo simulations along with Finite Element Models to evaluate the impacts of material property variations on structural dynamics in order to tackle these issues. By creating trustworthy probabilistic models that take these uncertainties into account, the ultimate goal is to improve the safety and dependability of structures. This will improve the design and analysis of composite structures that can withstand environmental factors and operating demands. This work makes the assumption that these models can accurately predict dynamic behavior, resulting in structural designs that are safer and more effective.*

KEYWORDS: *Monte Carlo Simulation, Natural Frequencies, Sensitivity Analysis, Uncertainty Quantification.*

1. INTRODUCTION

Quantifying uncertainties in material properties and their impact on vibration characteristics is crucial in structural and material analysis, especially in industries like aerospace. These uncertainties stem from factors such as variations in manufacturing processes, environmental conditions, and material degradation over time. Changes in properties such as Young's modulus and density directly affect the natural frequencies of a structure, thereby altering its dynamic response.

Young's modulus defines material stiffness, crucial for structural integrity, while density influences mass, impacting dynamic responses. Variations in these properties result in distributed rather than singular natural frequency values, introducing variability in vibrational behavior. Moreover, uncertainties in ply angle in composite materials can further influence vibration modes, affecting their shapes and amplitudes due to changes in structural rigidity and mass distribution. These uncertainties necessitate probabilistic modeling to accurately predict and mitigate potential impacts on structural dynamics.

To address these variations effectively, probabilistic methods and sensitivity analyses are employed. Techniques such as Monte Carlo simulations and global sensitivity analysis are instrumental in evaluating how variations in material properties impact vibrational responses. Moreover, Finite Element Models incorporating stochastic material properties enable comprehensive assessment by propagating uncertainties throughout the structural model.

The urgent need to increase the safety and dependability of structures in industries like aerospace and aviation, where failures can have disastrous effects, is what spurred this research. Uncertainties cannot be disregarded due to practical concerns such as material aging, differences in qualities caused by operating conditions, and production discrepancies.

The dynamic behavior of structures is impacted by these uncertainties, which can lead to unpredictable and potentially hazardous performance. Creating reliable probabilistic models to represent these uncertainties and improve the accuracy of vibrational behavior predictions for composite structures is the aim of this research. It is anticipated that the design and analysis of structures can be improved to guarantee they can successfully endure operational demands and environmental circumstances by putting the proposed approaches to use.

The underlying assumptions of this work are that natural frequencies and vibration modes of composite structures are greatly influenced by uncertainties in material properties, such as density and Young's modulus; that dynamic behavior caused by these uncertainties can be accurately predicted by probabilistic models, allowing for safer and more effective design; and that the application of the methodologies developed in this work will allow for the optimization of composite structures, minimizing risks associated with unforeseen variations in material properties.

2. MATERIALS AND METHODS

2.1 Propagation of uncertainties and Sobol indices

Let us consider a model $x \mapsto y = \mathcal{M}(x)$ and suppose that a probabilistic description of the input parameters is available in terms of a random vector X. The random response is defined by:

$$Y = \mathcal{M}(X) \tag{1}$$

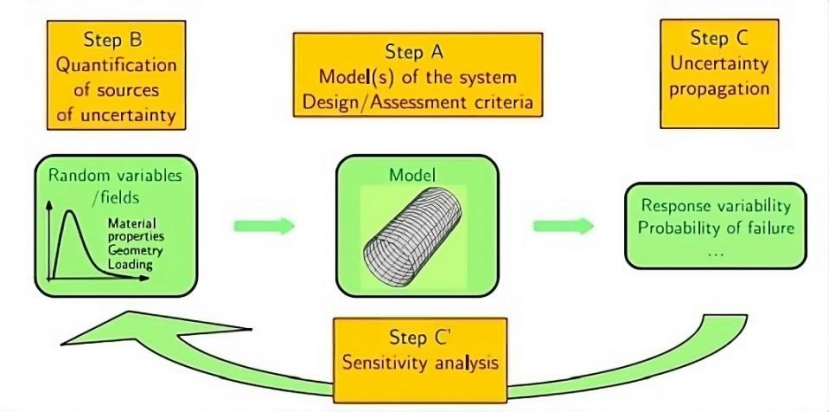
The main goal of probabilistic engineering mechanics is to study the probabilistic content of Y , namely its joint probability density function $f_Y(y)$.

There are two broad categories of uncertainty: epistemic uncertainty resulting from imprecision or lack of information, and aleatoric uncertainty arising from randomness. Epistemic uncertainty is a common obstacle encountered during the early stages of structure design, as it is not reducible (Wehrle, 2015).

The current state of ambiguity around composite materials may provide significant challenges throughout the design and manufacturing stages of the structural layers. There are several ways in which these uncertainties might be introduced into the system, including temperature sensitivity, hygroscopic phenomena, and the manufacturing process itself. Excessively conservative designs may result from inaccurate material properties, which are necessary to forecast failure mechanisms, dynamic behavior, and deformation (Bührer *et al*. 2021).

Uncertainty propagation methods usually provide information on the respective impact of the random input parameters on the response randomness, s shown in Figure 1. Such a hierarchization of the input parameters is known as sensitivity analysis, wich aims at selecting the important parameters, usually by defining quantitative importance measures or sensitivity indices that allow the analyst to rank the latter.

FIGURE 1: General sketch for uncertainty analysis



Source: Sudret (2007).

A good state of the art of the techniques is available in Saltelli *et al*. (2000), who gather the methods into two groups: regression-based methods and variance-based methods, this latter method is sometimes called ANOVA techniques for "ANalysis Of VAriance". Sobol indices allow for precise identification of the impact of each variable, decomposing total variance into parts attributable to individual variables and their combinations. This global, multidimensional evaluation is essential for understanding complex, nonlinear relationships between variables. The Sobol' indices are practically computed using Monte Carlo simulation, which makes them hardly applicable for computationally demanding models, e.g. finite element models in engineering mechanics.

The Sobol' indices are defined as follows :

$$S_{i_{1\ldots}\,i_s} = D_{i_{1\ldots\ldots\ldots i_s}}/D \tag{2}$$

Thus each index $S_{i_{1\ldots i_s}}$ is a sensitivity measure describing which amount of the total variance is due to the uncertainties in the set of input parameters $\{i_1, \ldots i_2\}$. The first order indices S_i give the influence of each parameter taken alone whereas the higher order indices account for possible mixed influence of various parameters. The total sensitivity indices S_{T_i} have been defined in order to evaluate the total effect of an input parameter (Homma and Saltelli, 1996). They are defined from the sum of all partial sensitivity indices $D_{i_{1\ldots i_s}}$involving parameter i:

$$S_{T_i} = \sum_{I_i} D_{i_{1\ldots\ldots\ldots i_s}}/D \qquad I_i = \left\{\left\{i_{1\ldots\ldots\ldots i_s}\right\} \supset \{i\}\right\} \tag{3}$$

$$S_{T_i} = 1 - S_{\sim i} \tag{4}$$

where $S_{\sim i}$ is the sum of all $S_{i_{1\ldots}\,i_s}$ that do not include index i.

2.2 Monte Carlo simulation

In this section, Monte Carlo simulation method for computing the mean value and standard deviation of a response quantity $Y = \mathcal{M}(X)$ are addressed.

Monte Carlo simulation can be used in order to estimate the mean value μ_Y and standard deviation σ_Y of a response quantity $Y = \mathcal{M}(X)$. Assume that a sample set of n input vectors has been generated, say $\{x^{(1)}, \ldots\ldots x^{(2)}\}$. The usual estimators of those quantities respectively read:

$$\hat{\mu}_Y = \frac{1}{n}\sum_{k=1}^{n} \mathcal{M}(x^{(k)}) \tag{5}$$

$$\hat{\sigma}_Y^2 = \frac{1}{n-1}\sum_{k=1}^{n}\mathcal{M}(x^{(k)} - \hat{\mu}_Y)^2 \quad (6)$$

In probabilistic engineering mechanics, the statistical values derived from sample sets are inherently random. It is typical practice to execute a single Monte Carlo simulation, often with a large sample size, to generate a single instance of the estimator. However, a rigorous application of Monte Carlo simulation should ideally include confidence intervals to provide a more comprehensive understanding of the results.

A rigourous use of Monte Carlo simulation should provide confidence intervals on the results though. Suppose that the sample size n is fixed by the analyst. The estimator is asymptotically Gaussian due to the central limit theorem. Thus the $(1-\alpha)$ confidence interval on $\hat{\mu}_Y$ reads, provided n is sufficiently large:

$$\hat{\mu}_Y - \mu_{\alpha/2}\hat{\sigma}_Y/\sqrt{n-1} \leq \mu \leq \hat{\mu}_Y + \mu_{\alpha/2}\hat{\sigma}_Y/\sqrt{n-1} \quad (7)$$

Where $\mu_{\alpha/2} = -\Phi^{-1}(1 - \alpha/2)$.

Alternatively, the latter equation may be used together with a criterion on the width of the confidence interval. Suppose that the mean value is to be computed with a relative accuracy of $\pm\varepsilon_{\mu_Y}$ with $(1-\alpha)$ confidence. Then the minimal sample size n_{min} is:

$$n_{min} = 1 + ceil[(\mu_{\frac{\alpha}{2}} CV_Y/\varepsilon_{\mu_Y})^2] \quad (8)$$

Where ceil $[x]$ rounds x upwards to the nearest integer and $CV_Y = \hat{\sigma}_Y/\hat{\mu}_Y$ is the estimated coefficient of variation of Y. This minimal number of samples n_{min} is inversely proportional to the square of the relative accuracy ε_{μ_Y} and proportional to the square of CV_Y .

2.3 Flexural Stiffness

At present, the determination of natural oscillation frequencies is commonly achieved through finite element displacement analysis, though less frequently through mixed methods or analytical approaches. However, to address the challenge of calculating flexural stiffness from dynamic structural tests, there is a need for a simpler method that directly correlates natural oscillation frequencies with the system's stiffness (Nesterenko, 2018).

Finite element modeling enables the representation of a mechanical system's mass as a collection of single-degree-of-freedom systems, each subject to boundary conditions specified by neighboring finite elements. For line continuum under flexural deformation pure bending, buckling and vibration the general governing differential equation is:

$$\frac{d^4W(x)}{dx^4} + \frac{N}{EI}\frac{d^2W(x)}{dx^2} - \frac{q}{EI} + \frac{m\gamma^2}{EI}W(x) = 0 \quad (9)$$

For pure bending analysis, the governing differential equation reduces to:

$$\frac{d^4W(x)}{dx^4} - \frac{q}{EI} = 0 \quad (10)$$

For buckling, the equation becomes:

$$\frac{d^4W(x)}{dx^4} + \frac{N}{EI}\frac{d^2W(x)}{dx^2} = 0 \quad (11)$$

When free vibration problem is encoutered, the governing differential equation becomes:

$$\frac{d^4W(x)}{dx^4} + \frac{m\gamma^2}{EI}W(x) = 0 \quad (12)$$

3. RESULTS AND DISCUSS

In this work, a plate of dimensions $a = 300\ mm$ and $b = 200\ mm$ and total thickness of $1.52\ mm$ is analyzed. The laminate stacking sequence is $[0^{\circ}/90^{\circ}]_{2S}$. This lay-up avoids bend-twist coupling $(D_{16} = D_{26} = 0)$, simplifying the problem. The elastic properties are given in the Table 1, where E_1 is the modulus of elasticity parallel to fibers; E_2 is the modulus of elasticity transverse to fibers; v_{12} is the Poisson's ratio, G_{12} is the in-plane shear modulus.

TABLE 1. Lamina mechanical propierties

$E_1(MPa)$	$E_2(MPa)$	v_{12}	$G_{12}(MPa)$	Thikness(mm)
125450	9450	0,32	4700	0,19

Source: Arakaki (2016).

According to NASA/TM—2006-214660, the distribution of each of the variables to be used is established in the Table 2 as follows:

TABLE 2. Probability distribution

$E_1(MPa)$	$E_2(MPa)$	$Density(^{kg}/_{m^3})$	$Angle\ ply(\ ^{o})$
Normal	Normal	Weibull	Normal

Source: NASA/TM—2006-214660 (2006).

The results obtained, which are presented in Table 3 indicate that to determine the natural frequencies of a plate under simple support conditions, the flexural rigidity matrices method provides a result very close to that obtained through a finite element analysis conducted using FEM software, as detailed below:

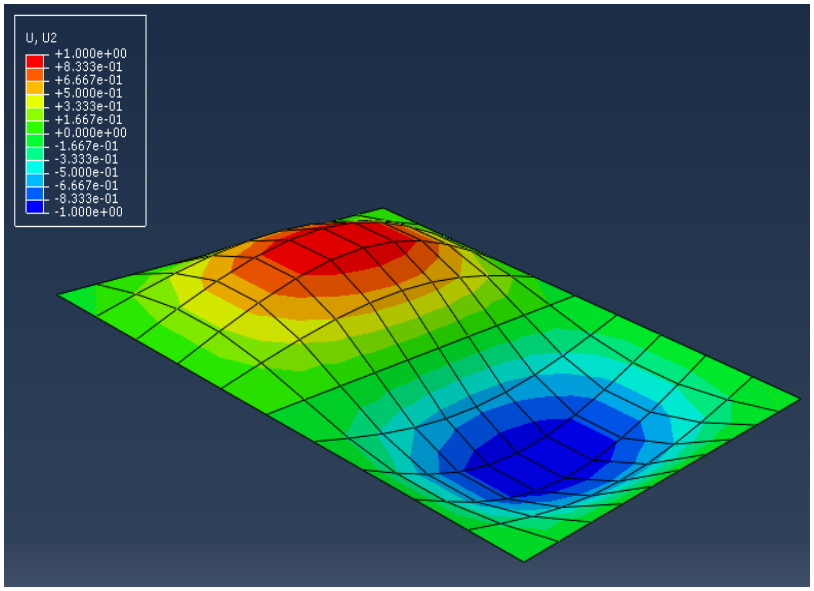
TABLE 3. Natural frequencies

	$\omega_{n1}(Hz.)$	$\omega_{n2}(Hz.)$	$\omega_{n3}(Hz.)$	$\omega_{n4}(Hz.)$	$\omega_{n5}(Hz.)$
Flexural rigidity matrices	87,08	201,25	249,27	348,28	399,80
FEM	77,39	191,38	251,28	317,09	426,65
% variation	11%	4,9%	0,8%	8,9%	6.7%

Source: Autors (2024).

In the plots made by python programming, as well as by FEM software, which are presented in Figure 2, it can be determined that there is a correlation in the displacements as a function of the properties of the plate.

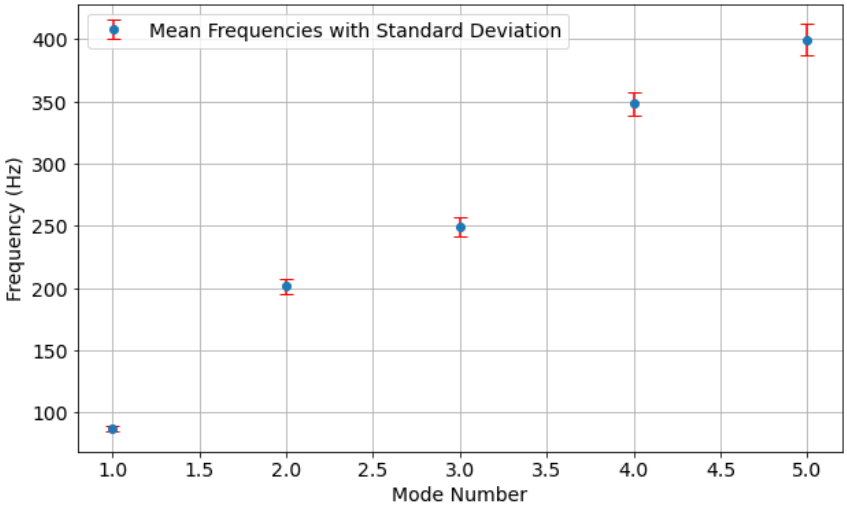
FIGURE 2: Results software FEM



Source: Autors (2024).

The plots of natural frequencies with error bars (standard deviations), which are presented in Figure 3 indicate substantial variation in the calculated frequencies due to uncertainties in material properties. The average natural frequencies show that while general trends can be observed, variations in material properties such as elastic moduli and fiber angles significantly affect the dynamic responses of the plate.

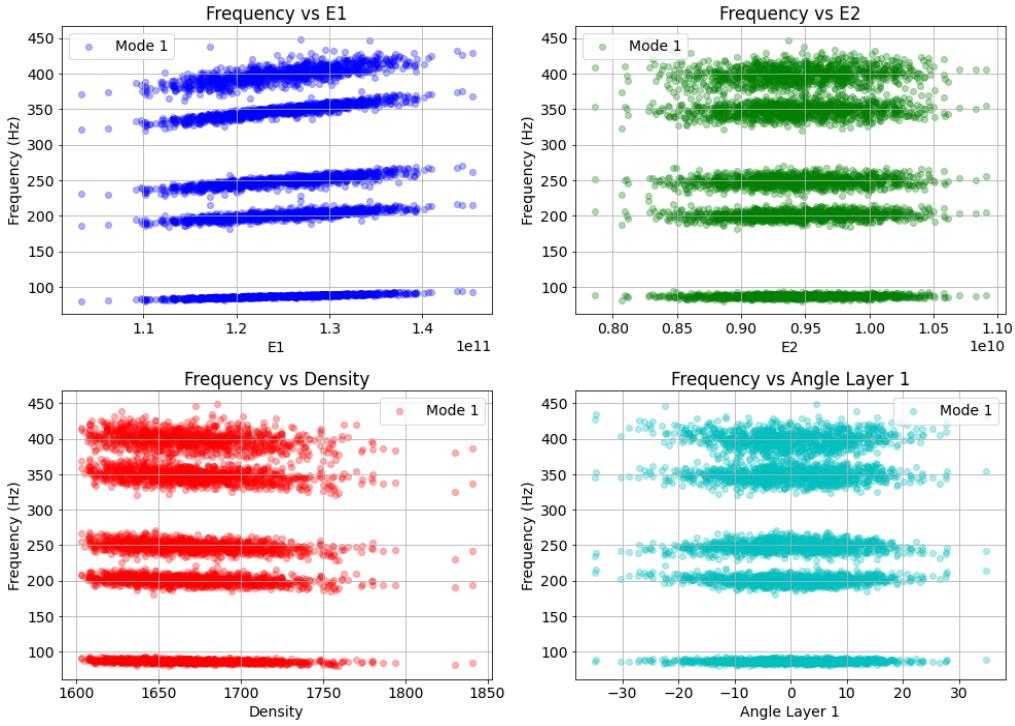
FIGURE 3: Natural Frequencies and Standard Deviations



Source: Autors (2024).

The graphs illustrating the relationship between each variable (elastic moduli, density, fiber angles) and natural frequencies, which are presented in Figure 4 reveal differential impacts on the modes frequencies. Stronger correlations were found with certain variables, suggesting that some properties have a more pronounced effect on the natural frequencies than others.

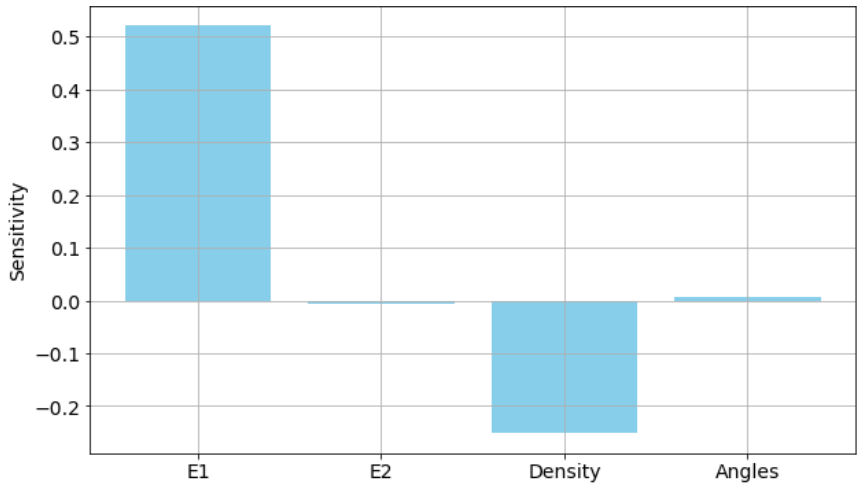
FIGURE 4: Impact of Material Parameters



Source: Autors (2024).

The scatter plots, which are presented in Figure 5 depict the relationship between the natural frequencies of the composite plate and variations in its material properties, specifically Young's modulus and density as well as the fiber angle of the first layer. Sensitivity analysis, employing Sobol indices, demonstrates that variability in material properties has a considerable impact on the natural frequencies of the plate. This analysis highlights which variables most significantly influence the natural frequencies, emphasizing their critical role in the design and optimization of composite structures.

FIGURE 5: Frequency Sensitivity / Sobol indices



Source: Autors (2024).

4. CONCLUSIONS

The Monte Carlo simulations illustrate that the natural frequencies of the composite plate are subject to considerable variation due to uncertainties in material properties and layer angles. The observed standard deviations highlight that while the average frequencies offer a useful approximation, actual frequencies can differ substantially. This variability should be taken into account in engineering design to ensure the plate functions consistently under diverse conditions.

The scatter plots show that the natural frequencies of the composite plate are highly dependent on variations in the density and Young's moduli of the material as well as the fiber angle. This result emphasizes how crucial it is to precisely regulate these parameters in the design stage in order to fine-tune the plate's vibrational characteristics and guarantee that it satisfies the intended performance standards.

The calculation of the natural frequencies by the flexural rigidity matrices method has an approximation of 6.46% on average to the one performed by FEM, so it would allow to estimate in an analytical way the natural frequencies in each mode.

The results of the sensitivity analysis show that natural frequencies are affected to varying degrees by several variables, such as density, fiber angles and Young's moduli. There are certain variables that significantly affect frequency changes more than others, the higher the Young's modulus the natural frequencies have a tendency to increase with a sensitivity of 0,52, while the higher the density the natural frequencies tend to decrease with a sensitivity of - 0,22.

The findings suggest that engineering designs should account for the potential variability in material properties and layer angles. Implementing robust design strategies and specifying tolerances can help manage these variations effectively. Additionally, focusing on the most sensitive variables

can guide optimization efforts, ensuring that the plate performs reliably according to design specifications and operational requirements.

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