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# HYPERELASTIC MODELS COMPARISON FOR MARITIME FENDERS

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## ABSTRACT

*Maritime fenders are part of structural elements that assist vessels in safe docking, playing an extremely important role. If damage occurs to the defenses, operating activities on the pier will be suspended, resulting in significant economic losses. Accurate estimation of docking energy is an important issue for sizing and selection of rubber fenders. Rubber's highly nonlinear stress-strain relationships make it difficult to model its mechanical behavior. The response curves of these elastomer materials are obtained through a series of experiments, such as tensile and compression uniaxial tests. Several constitutive equations for hyperelastic materials are presented in the literature, the main characteristics of which are high magnitude elastic deformation, without a plastic regime. In this work, reaction force versus displacement and strain energy versus displacement curves were obtained from a numerical finite element model of a rubber marine fender subjected to a unidirectional load. To perform this analysis, the Ansys software and two constitutive models, Neo-Hookean, Polynomial, Arruda Boyce and Mooney-Rivlin, were used. Part of these models represented adequately the force-displacement behavior of the fender in the 3D finite element simulation.*

**Keywords:** *Rubber fender, Hyperelasticity, Constitutive model, Force reaction, Strain energy.*

## INTRODUCTION

Docking structures enable the efficient and safe docking and mooring of vessels that visit ports, enabling the loading and unloading of goods and the embarkation and disembarkation of passengers, as well as their maintenance. When dealing with cargo movement, transport costs can be reduced by using larger ships, consequently, requesting a larger draft [1,2]. These specific characteristics impose adaptations and expansions in ports, which leads to construction in more environmentally challenging conditions, so that the loads and requests in maritime constructions and moored vessels will be greater and more complex. Most of these efforts occur when a vessel approaches the pier, at which point it is important to dock the vessel as smoothly as possible [3]. Despite this, large forces are exerted on the fenders and mooring mines that are generally fixed to the dock [4]. Fenders are essential structural elements for the protection of vessel berths, with the main objective of absorbing impact energy during berthing and constantly providing protection while they are moored. After mooring and when all mooring lines are secured, other efforts act on the vessels, such as: wind currents, movement and variation of the tide, loading and unloading of materials and inputs, these conditions produce kinetic energy in the fenders and mooring lines [4,5].

Rubbers are the most used materials for manufacturing marine fenders. To represent these materials, constitutive models applied to hyperelastic materials are used. Because rubber has a non-linear relationship along its stress-strain curve, exhibiting high magnitude elastic deformation, without plastic regime, making it impossible to use Hooke's Law to predict behavior of the material during its elastic deformation [6]. The constitutive models used to predict the behavior of this type of elastomers employ the concept of strain energy, establishing mathematical relationships between true tension and strain. Hyperelastic constitutive models can be divided into Micromechanical and Phenomenological. Each model represents a different approach. The first aim to give a better physical meaning to the constants used in strain energy functions. The second are based on experimental observations that make it possible to determine the constants present in the respective constitutive equations [7].
The present work proposes a static structural analysis of a fender used in one of the berths of the Itaqui Port, with the objective of characterizing the rubber based on a tensile test result. The fender analyzed was made of natural rubber reinforced with carbon black. To solve this problem, a nonlinear finite element analysis was carried out using the Ansys software. To characterize the material, some hyperelastic models from the software library were implemented from the test. Nonlinearities for large deformations and the hyperelastic behavior of the rubber material were considered.

## MATERIALS AND METHODS

### Design method of rubber fenders

To obtain a safe and secure berthing for the ship, there are limits to the acceptable movement and strength required for fenders, especially during liquid bulk transfer. For design purposes, it is important to predict all these loads and the necessary resistance of the fender, as accurately as possible [8]. Several international and national standards are available for this purpose [8-12]. When preparing a fenders systems project, it is essential to know the vessels for the port installation and their arrival frequency, in addition, knowledge of local conditions such as tide variation, wave incidence, wind currents and other meteorological factors is also important. Another criterion for a good fender design is the environmental and aggressive water conditions and the conditions of the loaded or unloaded vessel. Maritime fenders are built with the aim that their deformation energy ($E_d$) exceeds the effective energy ($E_f$) of mooring and berthing vessels, which is determined by the following expression:

$$E_f = \frac{1}{2} M V^2 C_m C_E C_c C_a F_s \qquad (1)$$

where, $M$ represents the ship's mass, $V$ ship's approach velocity, $C_m$ is the virtual mass coefficient, $C_E$ the eccentricity coefficient, $C_c$ dock configuration coefficient, $C_a$ damping coefficient, and $F_s$ safety factor. Determination of all these factors can be made using equations present in standards like [9].

### Hyperelastic constitutive models

Once the kinect energy of the ship is computed using Equation (1), it is necessary to select an appropriate fender model. To obtain the required energy dissipation, two main factors are crucial, the geometry of the specific model and the material properties of the rubber used. In this work a cylindrical geometry is considered. For material characterization, experimental data from a uniaxial tensile stress test was obtained in a stress-strain diagram [13]. In order to apply

those properties, which contain large displacements and highly nonlinear behavior, four constitutive models were used. For each of these theoretical models, parameters were calibrated using Ansys Workbench curve fitting specific capabilities.
Hyperelastic constitutive models are created, ideally, with the aim of describe or predict the complete behavior of the material. In order to do that, its uniaxial extension, biaxial extension and pure shear must be described accurately. These models can be classified as phenomenological or micromechanical based on how they approach their predictions. Four models were selected for this analysis and its correspondent equations are presented.
Rubber is assumed as having an isotropic elastic and nearly incompressible behavior. Its mechanical properties can be modelled through a Strain Energy Function, $U$, based on strain invariants, $I_1, I_2$ and $I_3$. Any stress strain analysis can then be performed using a function as

$$U = f(I_1, I_2, I_3) \tag{2}$$

**Polynomial Model**

Polynomial model is based on the first and second invariants $\bar{I}_1$ and $\bar{I}_2$ of deviatoric Cauchy-Green Tensor, with the Strain Energy Density, $U$, in incompressible form, given by

$$U = \sum_{i+j=1}^{N} C_{ij}(\bar{I}_1 - 3)^i(\bar{I}_2 - 3)^j \tag{3}$$

where, $\bar{I}_1$ and $\bar{I}_2$ are the first and second invariants of deviatoric strain, and $C_{ij}$ and $D_i$ are material constants. This model is convenient because one can choose the most suitable degree to the equation in order to better fit experimental data [13].
A reduced form can also be obtained considering that the strain energy varies significantly more in relation to the first invariant, mathematically it means that

$$\frac{\partial U}{\partial \bar{I}_1} \gg \frac{\partial U}{\partial \bar{I}_2} \tag{4}$$

Following this assumption, and using only terms in $\bar{I}_1$, Equation (5) is obtained

$$U = \sum_{i=1}^{N} C_i(\bar{I}_1 - 3)^i \tag{5}$$

**Neo-Hookean**

The reduced polynomial model, considering $N = 1$, becomes Neo Hookean model. This model, for incompressible materials is dependent only of the first deviatoric invariant, or distortional part of the right Cauchy-Green deformation tensor. The equation for this model then becomes,

$$U = C_{10}(\bar{I}_1 - 3) \tag{6}$$

Application of this model is adequate only for strains no greater than 30% to 40%.

**Mooney-Rivlin**

The model that has become known as Mooney-Rivlin is the polynomial model for $N = 1$, and for incompressible material has the simple form:

$$U = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) \tag{7}$$

where $C_{10}$ and $C_{01}$ are constants of calibration for different materials behavior.

**Arruda-Boyce**

These model and other physical models are based on an explanation of a molecular chains network. Strain Energy is assumed to be the sum of individual strain energies of individual chains oriented in space randomly. Equation for this model considering an incompressible material [6]

$$U = \mu \sum_{i=1}^{5} \frac{C_i}{\lambda_m^{2i-2}} \left(\bar{I}_1^i - 3^i\right) \tag{8}$$

where $C_1 = \frac{1}{2}$, $C_2 = \frac{1}{20}$, $C_3 = \frac{11}{21050}$, $C_4 = \frac{19}{7000}$, $C_5 = \frac{519}{673750}$, $\mu$ is the initial shear modulus and $\lambda_m$ is the locking stretch, defined as the value where upturn of the stress-strain curve rises significantly.

Finite Element Analysis

After a careful consideration of the constitutive models, a finite element simulation was carried out for the fender, in which a displacement of 60% of its length is imposed, in such a way that the geometry is compressed uniaxially along one of its principal axes. Such simulation, considers the possibility of contact and non-linearities coming for the large displacements and also due to the hyperelasticity of the rubber.

## RESULTS AND DISCUSSION

As discussed in previous sections, a cylindrical fender has been chosen for this study, measuring 1700 mm in length. Commercial Software Ansys Workbench was used to perform a finite element analysis. For this particular simulation, both ends of the fender were considered rigid, one of them is a fixed support and the other has been displaced for a distance equivalent to 60% (1020 mm) of the non-deformed length in the Z-direction. A multizone mesh with hexahedral linear elements was generated and is presented in Figure 1.

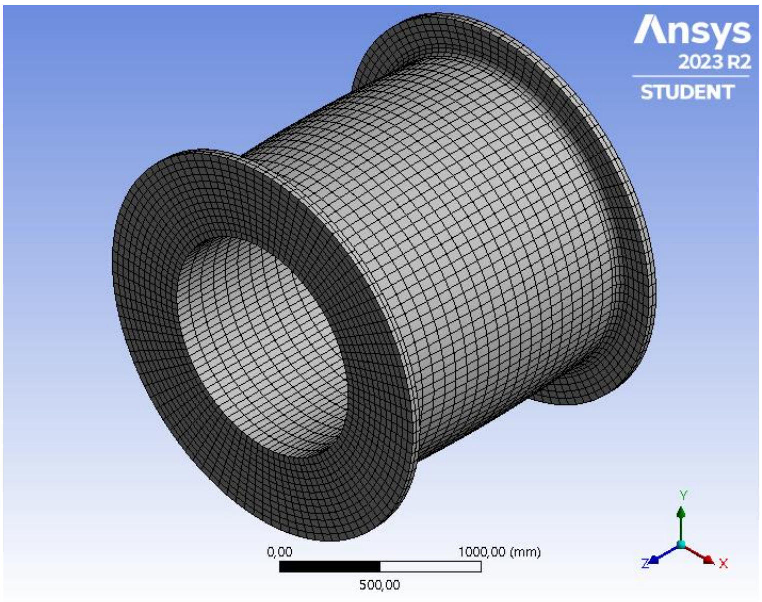



Fig. 1 – Three-dimensional geometry and mesh used for the finite element analysis performed in this work.

Mechanical parameters to characterize rubber behavior were obtained from data depicted in Figure 2. This plot shows stress-strain results from a uniaxial tensile stress test making evident the hyperelastic behavior and the large deformation of the specimen, up to 600%. Using this data, a curve fitting tool of Ansys Workbench was used to approximate this material behavior for more complex stress states, such as biaxial and shear stresses.

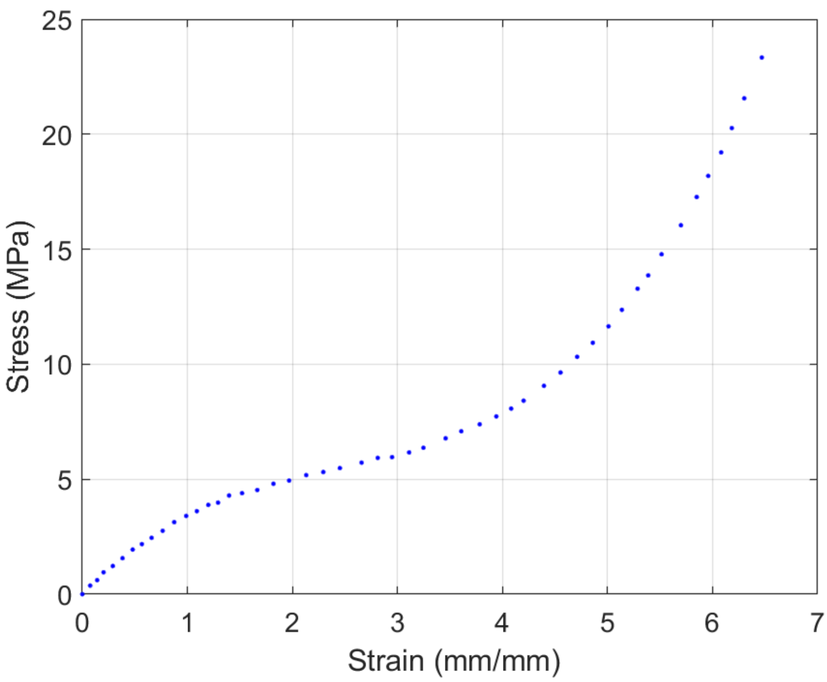
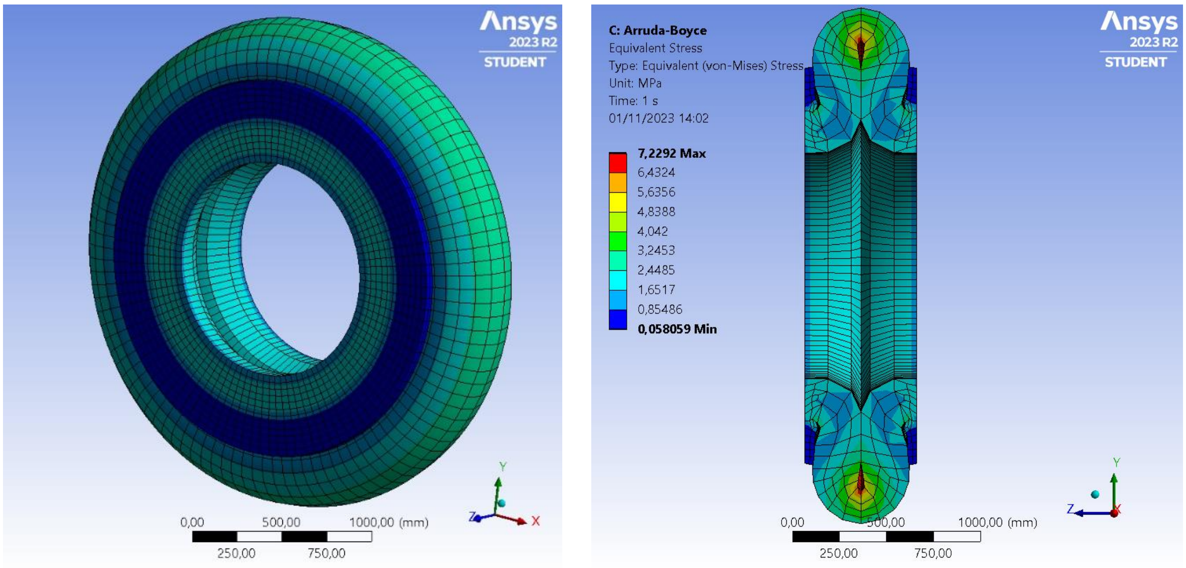



Fig. 2 – Stress Strain plot as a result of a uniaxial tensile stress obtained from [16].

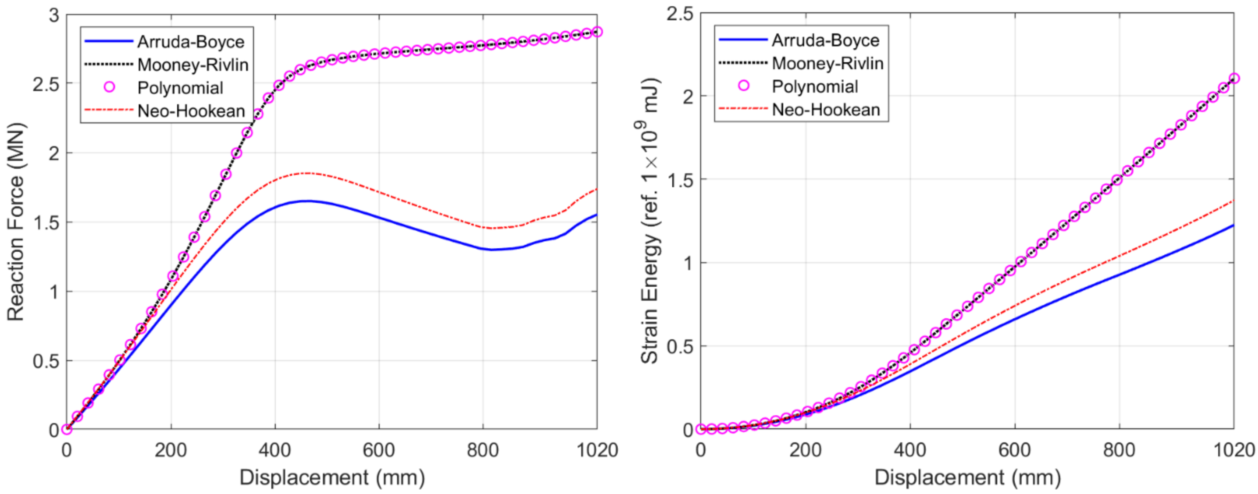
Figure 3 presents results obtained for the fender in its final deformed configuration. Figure 3b is a sectional view showing the results for the von Mises equivalent stress. The constitutive model considered in this simulation was the Arruda-Boyce. It can be seen that at this level of displacement the fender contacts itself internally.




a) b)

Fig. 3 – Deformed configuration of the fender in the extreme of displacements imposed showing distribution of the equivalent Von Mises stresses.

To assess the capabilities and accuracy of different material models in replicate the performance of the fender as described by its vendors in technical catalogues, results of reaction force vs. displacement and strain energy vs. displacement are presented in Figures 4a and 4b. In Figure 4a, variation of reaction forces at the fixed support, in Z-direction are presented as a function of the displacement at the same direction. Figure 4b shows the variation in strain energy as a function of displacement in the same direction.




a) b)

Fig. 4 – Results of the reaction force (a) and strain energy (b) as a function of the displacement imposed to the fender considering different constitutive models.

## CONCLUSIONS

A finite element analysis of a cylindrical rubber fender was successfully performed. Different material constitutive models were applied to describe their hyperlastic mechanical behavior. Comparisons of the results obtained and the performance curve observed in the manufacturers' manuals indicated that the polynomial and Mooney-Rivlin models were unable to reproduce reaction forces and variation in the fender's deformation energy when subjected to progressively greater uniaxial compression.

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