

THE INFLUENCE OF SEQUENTIAL LOADING IN RESIDUAL STRESS GENERATION

Guilherme Bordallo Estevam ⁽¹⁾ (guibordallo@gmail.com), Lucas Ricardo Carneiro Medina ⁽¹⁾ (lucasrcmedina@gmail.com), Danielle Lopes Zaidan ⁽²⁾ (danielle.zaidan@aluno.cefet-rj.br), Paulo Pedro Kenedi ⁽¹⁾⁽²⁾ (paulo.kenedi@cefet-rj.br)

⁽¹⁾ CEFET/RJ; Departamento de Engenharia Mecânica ⁽²⁾ Programa de Pós-Graduação em Engenharia Mecânica e Tecnologia de Materiais - PPEMM - CEFET/RJ

ABSTRACT: *Mechanical structures, under the influence of external loads, can develop cross section partial yielding, which in turn can generate residual stresses. Also, when the structure's operation stresses are combined with residual stresses, it can lead to "unexpected" premature failures. For this reason, it is important to understand how these residual stresses are generated, and what can be done to minimize their values. To access the main variables that could influence the residual stress cross section distribution, a numerical approach was implemented. The Finite Element Method was utilized to simulate several combinations of loading in two cycles: with positive or negative curvatures, with two loading levels and allowing or not the spring back. The combinations were applied on a metallic wire structure. For each case, the graphs Stress versus Strain, and the residual Stress distribution versus the wire cross section thickness, were provided. In function of a simple variable recombination, the preliminary results show a very different residual stress distribution. The variable combinations, which can diminish the residual stress cross section distribution, were also presented and discussed.*

KEYWORDS: *Residual stress, Sequential effects, Finite element model.*

A INFLUÊNCIA DO CARREGAMENTO SEQUENCIAL NA GERAÇÃO DE TENSÃO RESIDUAL

RESUMO: *Estruturas mecânicas, sob a influência de cargas externas, podem desenvolver escoamento parcial da seção transversal, o que por sua vez pode gerar tensões residuais. Além disso, quando as tensões de operação da estrutura são combinadas com tensões residuais, podem levar a falhas prematuras "inesperadas". Por esse motivo, é importante entender como essas tensões residuais são geradas e o que pode ser feito para minimizar seus valores. Para acessar as principais variáveis que podem influenciar a distribuição das tensões residuais na seção transversal, foi implementada uma abordagem numérica. O Método dos Elementos Finitos foi utilizado para simular várias combinações de carregamentos em dois ciclos: com curvaturas positivas ou negativas, com dois níveis de carregamento e permitindo ou não o "spring back". As combinações foram aplicadas em uma estrutura de arame metálico. Para cada caso, foram fornecidos os gráficos Tensão vs. Deformação e a distribuição de Tensão residual vs. espessura da seção transversal do arame. Uma simples alteração da ordem das variáveis mostra resultados preliminares muito diferentes na distribuição de tensões residuais. As combinações de variáveis que podem diminuir a distribuição das tensões residuais na seção transversal também foram apresentadas e discutidas.*

PALAVRAS-CHAVE: *Tensões residuais, Efeitos sequenciais, Modelo de elementos finitos.*

DECLARAÇÃO DE RESPONSABILIDADE

Os autores são os únicos responsáveis por este trabalho.

1. INTRODUÇÃO

Many studies have been realized about residual stresses estimation, with several approaches, as in Riagusoff *et al.* (2010), Zaidan and Kenedi (2019), Vignoli and Kenedi (2019), Rody, Kenedi and Pacheco (2014), Vargas (2014) and Lopes (2013). The aim of this paper is to estimate the cross-section residual stress distribution, of a bar of steel, after the imposition of two consecutive bending curvatures. The parameters used in this numerical model were the curvature signal (positive or negative), the load sequence (ascending or descending values) and if the spring back was or not allowed to happen. The main objective is to optimize the residual stress cross section distribution.

One direct application of these results would be, for instance, to contribute to the studies to prevent the failures observed on flexible pipes during the 21st century, when the phenomenon of CO₂ Stress Corrosion Cracking (SCC-CO₂), began to be noticed. In Fig. 1.(a) shows some initiatives to understand and mitigate the SCC-CO₂ failure. The principal objectives are minimizing the water ingress inside the annulus region and diminish the material susceptibility to the SCC-CO₂ phenomenon. Also, measuring and reducing the residual stresses distributions. As petroleum CO₂ and water content is not manageable, and the riser material improvements cannot be implemented in a fast way, the minimization of residual stresses seems to be a reliable way of mitigating this mode of failure for flexible risers and manifolds, as in Bai (2005). Fig. 1.(b) show an example of the flexible riser layers. The steel bar used in the paper's numerical example had dimensions and mechanical properties compatible with a typical tensile armour wire.

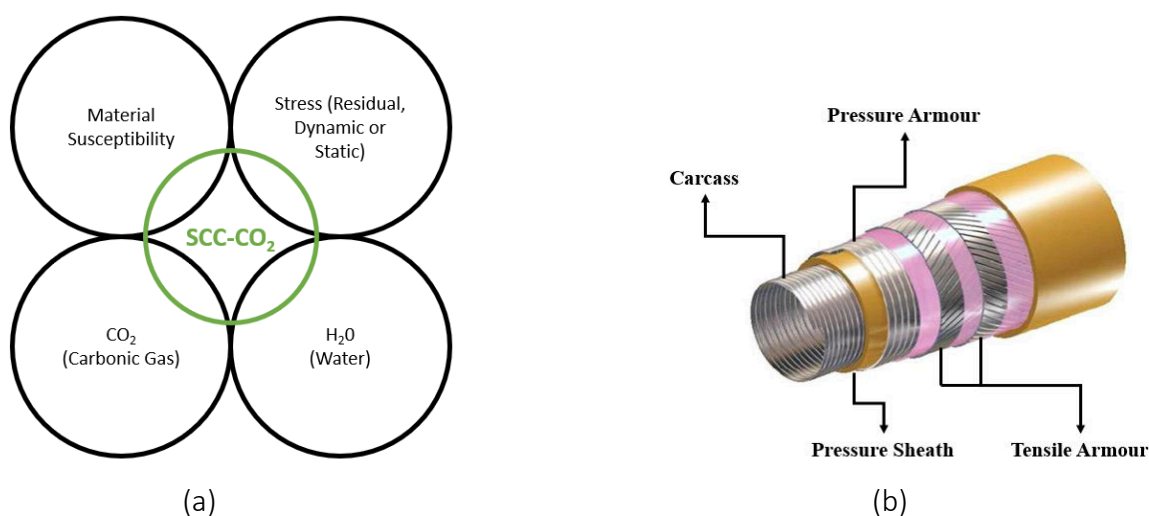


FIGURE 1. (a) Stress corrosion cracking initiation factors and (b) Main flexible riser layers. Source: Own Authorship. (2021)

2. BASICS

Residual stresses are presenting in manufacturing process, with magnitudes that are often a high fraction of the material yield resistance, as cited by Chuvas (2012) and James (2007). The ASM (2002) states that residual stresses are those that remain in mechanical components after external loads and thermal gradients have ceased. In fact, residual stresses are algebraically added to the operational load, which can significantly increase it. To determine the distribution of residual stresses through numerical analyses, several parameters are needed to be determined. Due to it, in this section, a basic knowledge of the residual stress, necessary to support the methodology used to implement the numerical analyses, are discussed.

It is known that residual stress values are influenced by different parameters. For this study, three input variables were considered: loading intensity, bending orientation, and presence or not of the spring back. The pure bending loadings, that were used, can be executed through the imposition of a radius of curvature smaller than the yielding radius of curvature (ρ_y), which is the curvature radius that the structure begins to yield. The equations shown in this section were obtained for an elastic perfectly plastic material, as in Estevam and Medina (2021), de Castro *et al.* (2020), de Castro *et al.* (2019) and Castro and Meggiolaro (2016). The bending moment that the structure, with rectangular cross-section, begins to yield (M_y) and the bending moment that the structure ends to yield (M_p) are show in Equation 1. Note that if M_p is reached, a plastic hinge can be formed in the cross section.

$$M_y = \frac{2}{3}bc^2S_y \quad ; \quad M_p = bc^2S_y \quad ; \quad f = \frac{M_p}{M_y} \quad (1)$$

Where b is the width, c is the semi-thickness of cross section and S_y is the yielding resistance. Equation 2 shows a bending moment, which yield partially the rectangular cross-section, for $M_y < M(\rho) < M_p$.

$$M(\rho) = \frac{3}{2} \left[1 - \frac{1}{3} \left(\frac{y_y}{c} \right)^2 \right] M_y = \frac{3}{2} \left[1 - \frac{1}{3} \left(\frac{\rho}{\rho_y} \right)^2 \right] M_y \quad ; \quad \text{where, } \rho_y = \frac{E}{S_y} c \quad \text{and} \quad \frac{y_y}{c} = \frac{\rho}{\rho_y} \quad (2)$$

Where y_y is the elastoplastic border, which separate the elastic and the plastic regions. The factor form f , of Equation 1, for a rectangular cross-section is 1.5.

The multiplier factor α , presented in Equation 3, which varies $1 \leq \alpha < f$, can be used to establish the different levels of loadings, as presented in Equation 4.

$$\alpha = \frac{3}{2} \left[1 - \frac{1}{3} \left(\frac{y_y}{c} \right)^2 \right] = \frac{3}{2} \left[1 - \frac{1}{3} \left(\frac{\rho}{\rho_y} \right)^2 \right] \quad (3)$$

$$M(\rho) = \alpha M_y \quad (4)$$

Two intermediate values of α were proposed, $\alpha = 1.2$ (less severe) and $\alpha = 1.4$ (more severe). Also, the curvature orientation (positive or negative) is an important input of the proposed model. As it will be seen in the numerical model results, it is completely different a curvature sequence with two positive curvatures and a curvature sequence with one curvature positive and the other negative. The stress distribution in cross-section, at loading phase, is represented by Equation 5.

$$\sigma_l(\rho, y) = \begin{cases} \text{const} \cdot (s_y), & \text{for } y_y < y \leq c \\ \text{const} \cdot \left(\frac{y}{\rho \cdot \varepsilon_y} \cdot s_y \right), & \text{for } -y_y < y \leq y_y \\ \text{const} \cdot (-s_y), & \text{for } -c < y \leq -y_y \end{cases} \quad (5)$$

Where $\varepsilon_y = S_y / E$. Note that the $\text{const} = -1$ for positive $M(\rho)$ and $\text{const} = 1$ for negative $M(\rho)$. After the relief of the loading $M(\rho)$, that partially yielded the cross section, there is an elastic return, named spring back, that contributes to the generation of the residual stress distribution. Equation 6 describes the spring back, as in Crandall, Dahl and Lardner (1978).

$$\sigma_{sb}(\rho, y) = \frac{-M(\rho)y}{I}, \text{ where } I = \frac{bh^3}{12} \quad (6)$$

Where I is the moment of inertia of area and y is a vertical distance, from the neutral line to a generic point. Although the application of Equation 6 is limited to the elastic range, the material has a doubled yielding limit to use after the first cycle of plasticity. So, the spring back occurs in completely elastic way. The residual stress distribution can be estimated by Equation (7):

$$\sigma_{res}(\rho, y) = \sigma_l(\rho, y) + \sigma_{sb}(\rho, y) \quad (7)$$

Where the indexes l , sb and res , means, respectively, load, spring back and residual. The Equation 7 is valid for a load/unload cycle. For more than one cycle, other concepts must be employed. In the sub-section model results, this fact will be commented. The numerical model is explained in the next section.

3. NUMERICAL MODEL

The numerical model was proposed with the utilization of ANSYS Mechanical Workbench software. It was supposed that wire was subjected to a plane stress state. The model details are presented in Table 1.

TABLE 1. Numerical model details.

Numerical model properties	Value
Type of Element	Quadratic
Number of Nodes	2821
Number of Elements	900
(See Appendix 1 for convergence analysis)	

Source: Own Authorship (2021).

The applied loading was imposed in two levels, using Equation (3) and Equation (4) and the cross-section dimensions of Table 2: $\alpha = 1.2$, that corresponds to the applied bending moment of $M = 95.04 \text{ N}\cdot\text{m}$ and $\alpha = 1.4$, that corresponds to the applied bending moment of $M = 110.88 \text{ N}\cdot\text{m}$. The curvature could be positive or negative. Also, the spring back can be allowed or not (by restraining the wire elastic return). Thus, for each cycle, there are eight different combinations, and therefore, for two cycles, there are a total of sixty-four combinations. Many of the combinations are said to be "mirrored" because symmetries. So, it was decided to show, in Table 3, only four of the most representatives of these combinations, encompassing almost all-important model contribution.

3.1 The model assembly

For the present research, the geometrical and mechanical properties of the metallic wire are shown in Table 2.

TABLE 2. Tensile armour wire geometrical and mechanical properties.

geometrical and mechanical properties	Value
Thickness, $2 \cdot c$ (mm)	6
Width, b (mm)	12
Length, L (mm)	Variable
E (GPa)	200
E_t (GPa)	0.5
S_y (MPa)	1100
Poisson ratio	0.3

Source: Own Authorship (2021).

Where E is the Young modulus. The tangent modulus E_t describes the material hardening in the plastic phase. It was introduced to enable the application of the numerical model. Thus, it was arbitrarily chosen a small value, to be comparable to the equations of the former section, which were proposed for an elastic perfectly plastic material. Table 3 presents the studied cases.

TABLE 3. The studied cases.

Case	Cycle 1			Cycle 2		
	α	Orientation	<i>Spring Back?</i>	α	Orientation	<i>Spring Back?</i>
1	1.2	Positive	Yes	1.4	Positive	Yes
2	1.2	Positive	Yes	1.4	Negative	Yes
3	1.4	Positive	Yes	1.2	Positive	Yes
4	1.4	Positive	Yes	1.2	Negative	Yes

Source: Own Authorship (2021).

Note that the spring back was always allowed to generate residual stresses after all cycles. The sequence of loading cycles is represented by Fig. 2.a. It was considered only half-length of the wire, due to the geometrical and loading symmetries presented by the model. The boundary conditions are shown in Fig. 2.b, positioned at the vertical symmetry plane. At point A, its movement was restricted in horizontal and vertical axes, at point B the movement was fixed only in the horizontal direction. The bending moment was applied, at the other end, at point C.

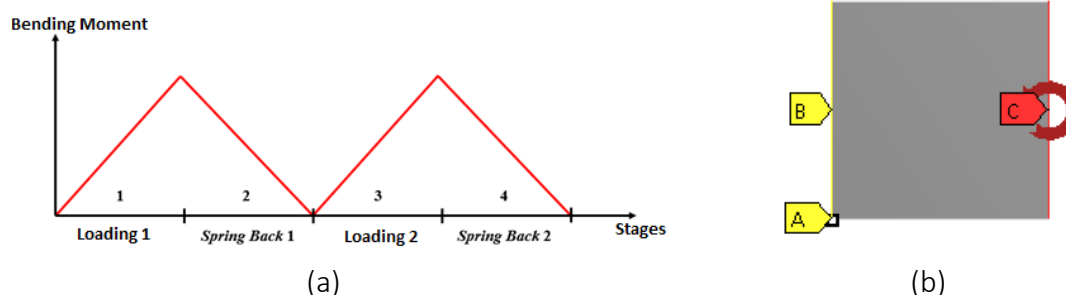


FIGURE 2. (a) Representation of the load cycles sequence and (b) The boundary and the loading conditions. (Note that the horizontal length is not in scale). Source: Own Authorship (2021).

To extract the distribution of residual stresses along the section, the 'Path' function of ANSYS Mechanical Workbench was used. It allows evaluating the distribution of residual stresses along a cross section vertical line of the metallic wire. The stress residual points were generated at the 'Path' line, after the fourth step of loading, as shown in Fig. 2.a. In sequence, the Origin software was used to graph the Residual Stress vs. Cross section vertical position (CSVP) graphics. The Stress versus Strain graphics, of a point present on the lower surface of the metallic wire during all load steps, are also generated by F.E. model file and graphed with the aid of the Origin software.

3.1 Model results

The model results are divided in two sets. The first set is composed by the cases 2 and 4. They were obtained through the application of the analytical approach, with the use of equations (1-7) and Tables 2 and 3, in a mathematical software, as Mathcad. They show, in a detailed way, how the residual stress distribution is formed through a sequence of Stress versus CSVP graphics. These graphics are shown in Fig. 3 and Fig. 4. In the second part, the numerical model results are presented for the cases 1, 2, 3 and 4, in two graphic types: Residual stress versus CSVP graphic and the Stress versus Strain graphic.

Fig. 3 shows every step of case 2 of Table 3, where the Stress versus CSVP graphics are presented for two subsequent cycles: First the wire is bending, with $\alpha_1 = 1.2$ and positive curvature. The spring-back is allowed. In sequence the wire is bending, with $\alpha_2 = 1.4$ and negative curvature. The spring-back is, also, allowed. In Fig. 3 case, with $\alpha_1 \leq \alpha_2$, the residual stress distribution of the cycle 1, shown at Fig. 3.c, was not added to the load stress distribution of cycle 2, shown in Fig. 3.d. When $\alpha_1 \leq \alpha_2$, the behavior was as the cycle 1 didn't exist at all, Castro and Meggiolaro (2016).

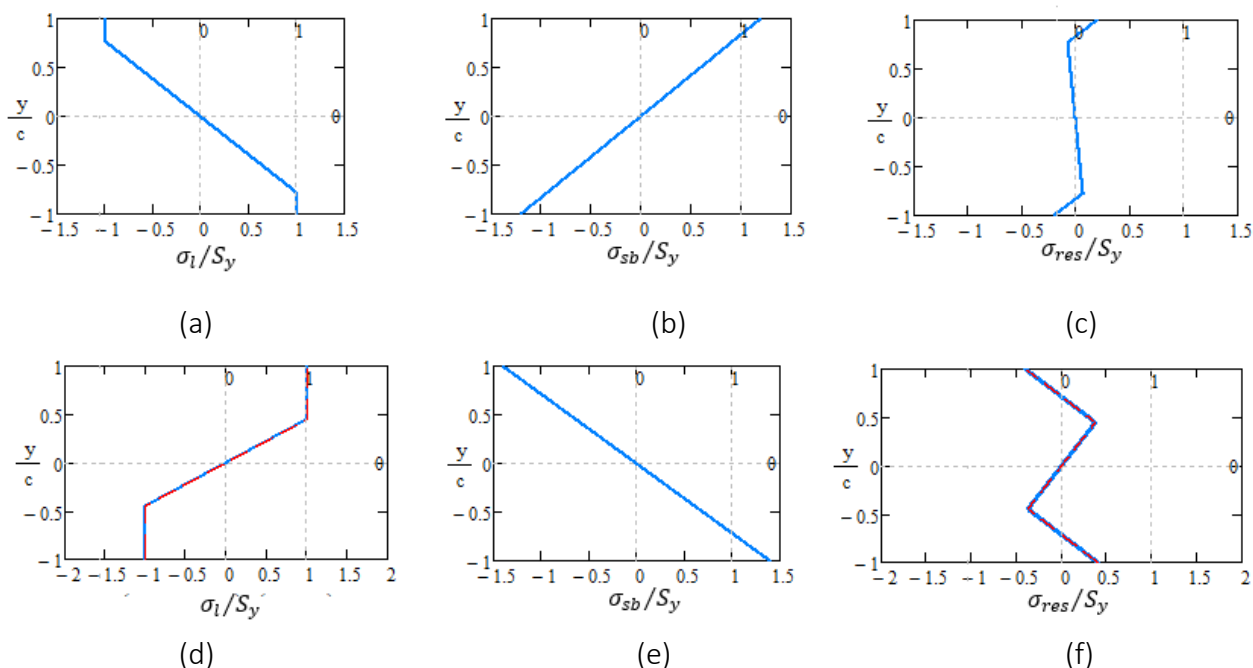


FIGURE 3. Stress versus CSVP of case 2 in the sequence: loading, spring-back and residual stress. First cycle: (a), (b) and (c). Second cycle: (d), (e) and (f). Source: Own Authorship.

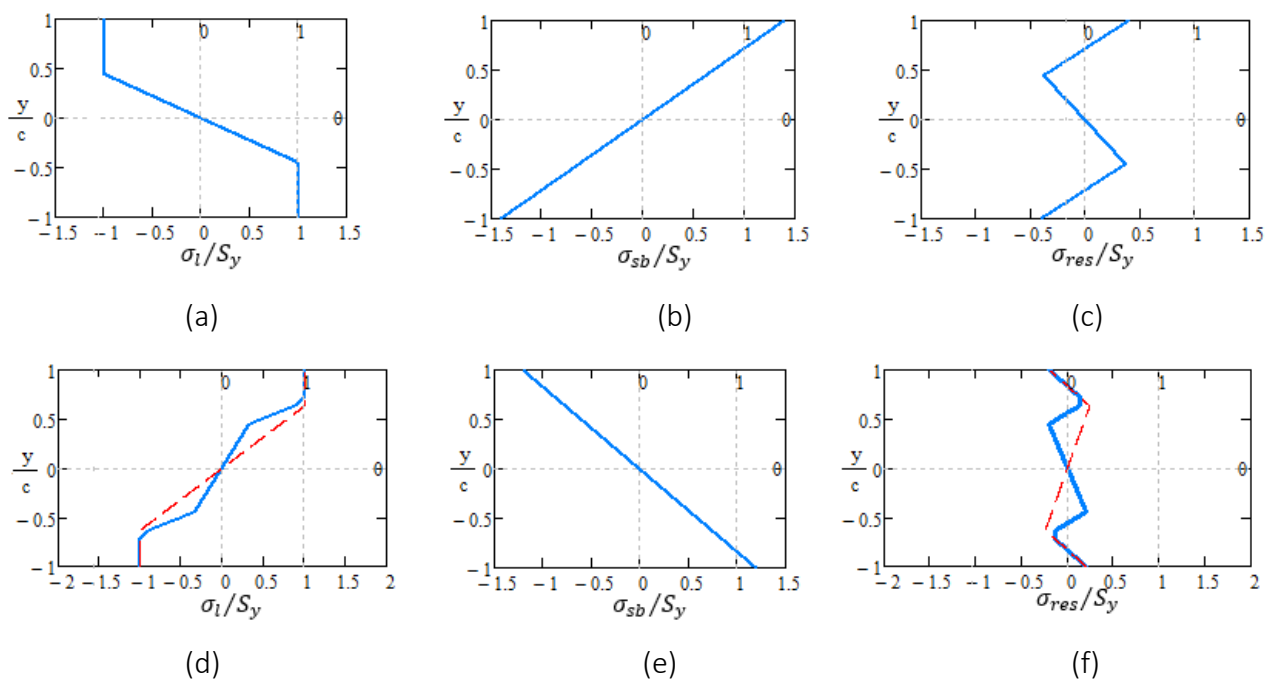


FIGURE 4. Stress versus CSVP of case 4 in the sequence: loading, spring-back and residual stress. First cycle: (a), (b) and (c). Second cycle: (d), (e) and (f). Source: Own Authorship.

Note that σ_l is the load stress distribution, σ_{sb} is the spring-back stress distribution and σ_{res} is the residual stress distribution.

Fig. 4 shows every step of case 4, of Table 3, where the Stress versus CSVP graphics are presented for two subsequent cycles: First the wire is bending, with $\alpha = 1.4$ and positive curvature. The spring-back is allowed. In sequence the wire is bending, with $\alpha = 1.2$ and negative curvature. The spring-back is, also, allowed. In this case of Fig.4, with $\alpha_1 > \alpha_2$, the residual stress distribution of the cycle 1, shown at Fig. 4.c, was added to the load stress distribution of cycle 2, shown in Fig. 4.d. Note, in this same figure, the red dashed line represents the load stress distribution if only cycle 2 was considered. On the other hand, the blue line represents the summation of the residual stress distribution of cycle 1 (Fig. 4.c) with the load stress distribution of cycle 2 (red dashed line of Fig. 4.d). In other words, when $\alpha_1 \leq \alpha_2$, the residual stress distribution of the previous cycle is added to the load stress distribution of the ulterior cycle, Castro and Meggiolaro (2016).

Analyzing Table 3 for cases 2 and 4, in both cases, the wire has first a positive orientation and then a negative orientation. In all cases the spring-back were allowed. Although both cases have submitted to the same loading, they were applied in a different sequence. In Case 2 it was first applied $\alpha_1 = 1.2$ and then $\alpha_2 = 1.4$, and in Case 4 it was first applied $\alpha_1 = 1.4$ and then $\alpha_2 = 1.2$. This sequence difference between these two cases generate completely different cross sections residual stress distribution, as can be seen comparing Fig. 3.f and Fig. 4.f. Also, the maximum residual stress generated by case 4 was half the maximum residual stress generated by case 2.

Figures (5-8) show the numerical model results. Each Figure has two different graphics. The (a) is composed by Residual Stress versus CSVP graphics and the (b) by Stress vs Strain graphics of a point localized at the wire bottom. Note that Figures (3-8) had their axis parameterized. The stress axes are divided by yielding strength S_y , the strain axes are divided by yielding strain ϵ_y and the vertical position axis y are divided by the wire semi-height c . Also note, that in Figures (5-8) the analytical approach is additionally shown, with red points. Fig. 3.f and Fig. 6.a, and the Fig. 4.f and Fig. 8.a correspond, respectively, for cases 2 and 4. The utilization of the Stress versus Strain graphics, to follow a particular point of the wire cross section, improve the understanding about the residual process evolution because show the stress and strain evolution in one graphic.

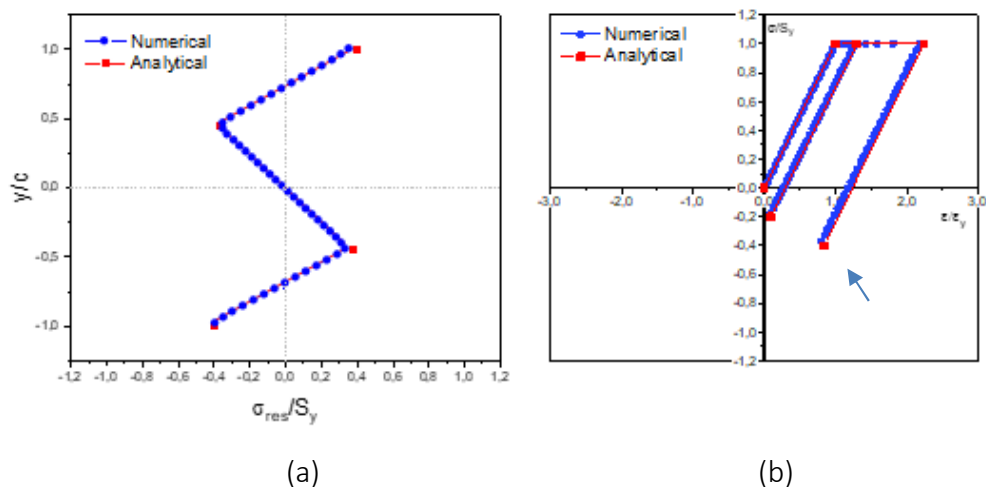


FIGURE 5. Case 1: (a) Residual stress vs. CSVP and (b) Stress vs. Strain. Source: Own Authorship.

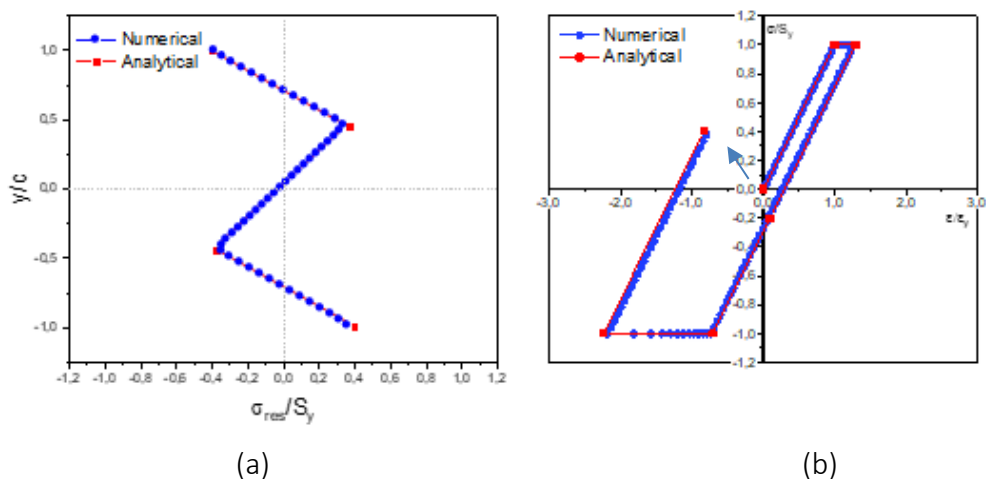


FIGURE 6. Case 2: (a) Residual stress vs. CSVP and (b) Stress vs. Strain. Source: Own Authorship.

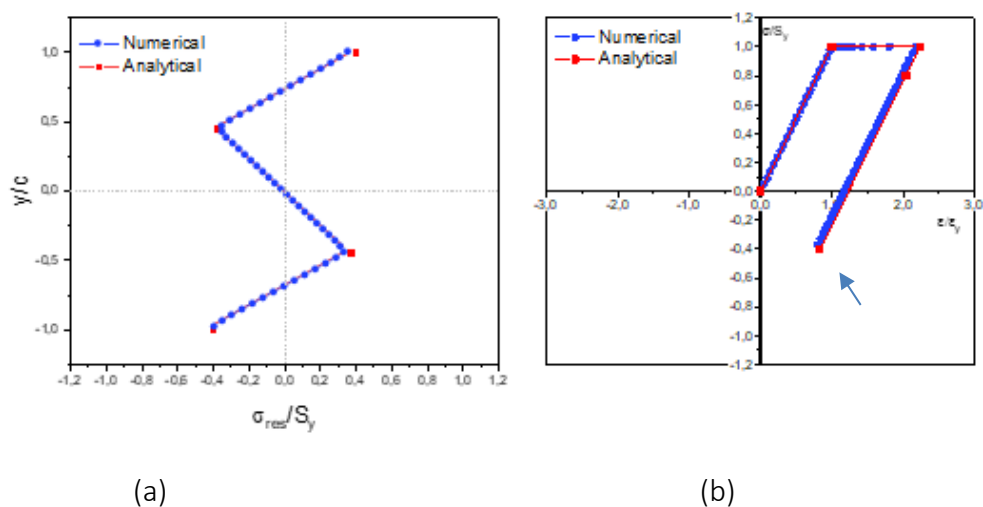


FIGURE 7. Case 3: (a) Residual stress vs. CSVP and (b) Stress vs. Strain. Source: Own Authorship.

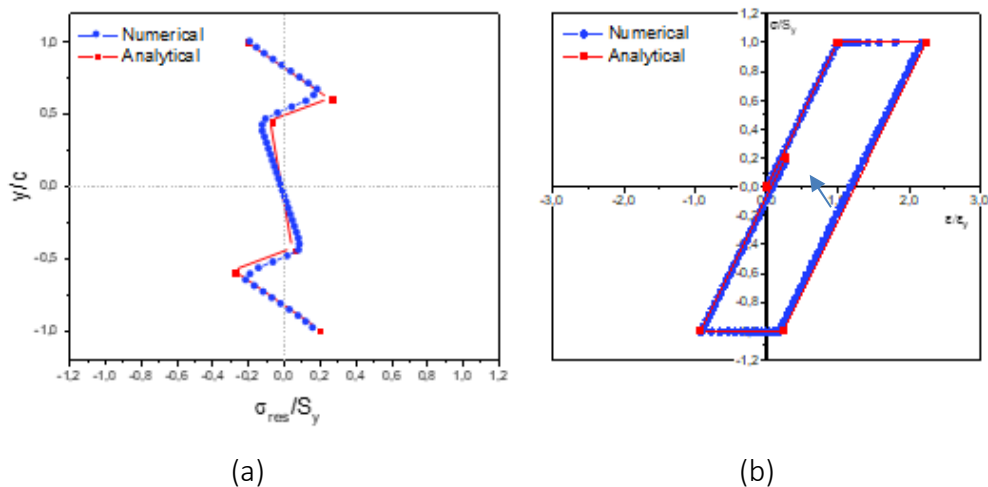


FIGURE 8. Case 4: (a) Residual stress vs. CSVP and (b) Stress vs. Strain. Source: Own Authorship.

This is quite interesting if you are trying to figure out what combination of sequential loadings minimize the residual stresses in a cross section specific point. For instance, comparing the final point (located by an arrow) of Figures 6.b and 8.b, it is clear that the final point of case 4 is closer to the axis origin, revealing lower residual stress and strains if compared with case 2.

4. CONCLUSIONS

This study assessed the residual stress distribution profiles in metallic wires, submitted to two sequential curvatures, by the utilization of a professional finite element software. After simulating, the numerical results show a great difference of the residual stress distribution after sequential curvatures. It was observed that for sequential wire bending loadings, of different sequential curvatures signs (in this example from positive to negative, but similar results would be obtained from negative to positive curvatures), produces significant differences in the final residual stress distribution. When $\alpha_1 \leq \alpha_2$, the final residual stress distribution results show as if only the second cycle was present. For the case with $\alpha_1 > \alpha_2$, the residual stress distribution of the previous cycle was added to the load stress distribution of the posterior cycle. In fact, adopting the case with $\alpha_1 > \alpha_2$, for different sequential curvature sign, in the paper example, the maximum residual stress value reached was only half the value of the case with $\alpha_1 \leq \alpha_2$.



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APPENDIX

1. Constitutive model and convergence

For the numerical model, it was considered that the material follows the elastic with hardening constitutive model. A tangent modulus of 0.5 GPa was selected to avoid problems in the convergence of the numerical model. Fig. A1.a shows the graphical representation of the numerical constitutive model and Fig. A1.b shows the convergence analysis.

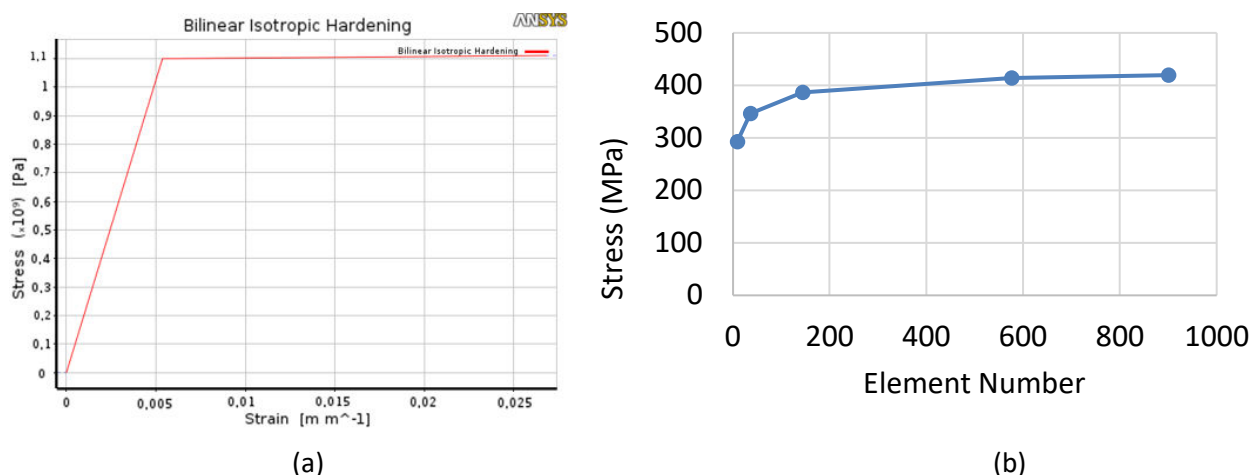


FIGURE A1. (a) Stress versus Strain graph for the material used in the model. Extracted from ANSYS. (b) Stress versus number of elements for the material used in the model. Source: Own Authorship.

Note that the tangent modulus adopted, the hardening phenomenon is quite small. For the present study, it was considered that the convergence was achieved when it was obtained a percentage difference of 5% between the stresses of the same point of the wire with different number of elements of the numerical model. After the convergence study, with the refined the mesh, a relatively small number of elements (900 elements) were obtained, because the wire simple geometry and its vertical symmetry.