Sraffa's criticisms in light of the debate between Garegnani (1970) and Samuelson (1962)

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Abstract

Despite Sraffa's (1960) critique of the marginalist theory of long-run general equilibrium, recent studies, based on estimates of wage-profit curves using input-output matrices, question the relevance of the "reswitching of techniques" and "reverse capital deepening", considering them to have a low probability of occurrence. These studies claim that the wage-profit curve is approximately linear. Because of this, there is an argument that Samuelson's (1962) surrogate production function could be rehabilitated. This essay aims to elucidate the problems of Samuelson's (1962) surrogate production function, criticized by Garegnani (1970) and Baldone (1984), demonstrating that its inconsistency frustrates any attempt to consider it a valid approximation, even given the quasi-linearity of those curves.

Keywords: Sraffa; Reverse capital deepening; Reswitching of techniques; Wage–profit curve; Production function.

Área 3. Método, Teorias Econômicas Heterodoxas e História Econômica

Introduction

Although the problem of the indeterminateness of capital endowment for marginalist long-period general equilibrium theory remains unchallenged since Sraffa's (1960) book to the present day, in more recent debates, the phenomena of "reswitching of techniques" and "reverse capital deepening" have been criticized for their presumed low likelihood of occurrence. Supposedly, this result is confirmed through estimates of input-output matrices from various countries and would imply the consistency of the marginalist view of capital as an approximation and a weakening of Sraffa's (1960) critique.

According to these estimates, wage–profit curves in several countries over different periods are approximately linear. This means that relative prices do not change much when the rate of profits rises from zero to its maximum level, as real wages fall, given the methods of production in use. This empirical regularity has been found by some authors, including Ochoa (1989), Han and Schefold (2006), Schefold (2013, 2020), Iliadi *et al*. (2014), Shaikh (2016, chapter 9), Mariolis and Tsoulfidis (2016, chapter 3), Shaikh, Coronado and Nassif-Pires (2020), Torres-González (2020), Kersting and

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Schefold (2021), Tsoulfidis (2021, chapter 5, 2022), Ferrer-Hernández and Torres-González (2022), among many others.

Regarding criticisms of the demand side, there have been further attempts to downplay the significance of reverse capital deepening and reswitching of techniques. One such argument concerns the low likelihood of these phenomena occurring. Han and Schefold (2006) and Schefold (2013, 2020, 2023) are representative of this argument. But it is not just this. Schefold (2013, 2016, 2022) also defends the approximate surrogate production function, his own "version" of the surrogate production function originally developed by Samuelson (1962) and criticized by Garegnani (1970). However, the latter function was forcefully criticized by Garegnani (1970) and Baldone (1984), with the latter author being supported by Salvadori and Steedman (1988) and D'Ippolito and Potestio (1993). Despite this, misunderstandings about economic theory persist in recent debates.

For that reason, it is necessary to clarify in the simplest way possible the critiques developed throughout Capital Controversies, specifically the discussion between Samuelson (1962) and Garegnani (1970). Therefore, the aim of this essay is to elucidate in the simplest way possible the problems of the surrogate production function, which frustrates any attempt to consider it as an approximation or as a useful "tool". Section 2 presents a simple model of an economy that contains only circulating capital and produces only two basic goods (corn and bread) to discuss in section 3 Samuelson-Garegnani debate. Section 4 incorporates both Baldone's (1984) critique of "non-switching" to the model presented in the previous section and further considerations on the Samuelson's (1962) model. Section 5 discusses the implications of the capital supply-side critique and the specificity of the surrogate production function. Finally, the final remarks.

2 A simple representative model of the Samuelson-Garegnani debate: an economy that contains only circulating capital and produces only two basic goods (corn and bread)

2.1 CAPITALIST ECONOMY WITH HETEROGENEOUS GOODS AND ONE TECHNIQUE

Assume a capitalist economy where there are two sectors producing a single commodity[2] each, corn and bread, and each sector has only one production process. Each production process lasts one year and requires corn and labor as inputs at the beginning of the production period. Constant returns to scale in each production process are assumed.[3] At the end of the period only a single commodity (corn or bread) in each sector emerges.

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[2] That is, there is no joint production.

[3] Note that if the model were to represent classical surplus theory, it would not be necessary assume constant returns to scale or any other assumption about variable returns, since that theory assumes a given output level. Here this hypothesis is assumed because, ultimately, the aim is to discuss marginalist theory and its problems, just as Garegnani (1970) discussed.

Thus, the first sector produces a single capital good, corn, and the second one produces a single consumption good, bread. There is no net investment, i.e., corn (capital good) is completely used up during the single year (the economy is in a stationary state). So, the economy's net output consists of an amount of consumption good. The capital good is a circulating one and there is no fixed capital. Moreover, there is only one type of labor (homogeneous labor), and wages are paid after the production process period in terms of units of consumption good.[4] Land and other natural resources are overabundant – therefore its respective remunerations are null.

This economy produces the corn necessary to produce itself and bread, which is necessary for the reproduction of labor force. In this way, it produces a positive net output consisting of consumption good. It is assumed that wages are entirely variable. However, by doing this, bread would not appear among the means of production and would cease to be a non-basic good. Thus, to circumvent this problem, it is assumed that bread is the only good contained in the workers' consumption basket and that the real wage consists of a quantity of bread. Moreover, surplus is a residually determined quantity and there is free mobility of capital, so the profit rate is uniform across all sectors. To represent these two production processes, let the numbers 1 and 2 denote commodities corn and bread, respectively. Both commodities appear in sector 1, which produces capital good, and sector 2, which produces the consumption good, correspondingly.

Denote as A_{ij} the quantity of commodity i required as means of production in sector j, $i, j = 1,2$, for producing X_j which is the quantity of commodity j, $j = 1,2$, arising at the end of production period. X_j is the gross output of sector j. Let L_j be the amount of direct labor required in sector j. Both these amounts are measured in physical units. In addition, denote a_{ij} as the quantity of commodity i required as a means of production and used up per unit of gross output of commodity j, where $i, j = 1,2$. In the analogous way, denote l_j as the quantity of labor j required as a means of production and used up per unit of gross output of commodity j, where $j = 1,2$.

In this economy, there is initially only one available technique (or production system), which refers to a set of methods of production (or production processes), one for each commodity. For example, corn and labor are necessary to produce capital good (corn), so the method of production of this good is given by the pair (a_{11}, l_1). And corn and labor are necessary to produce consumption good (bread), so the method of production of this good is given by the pair (a_{12}, l_1). So, there are two methods of production, one for each commodity: (a_{11}, l_1) and (a_{12}, l_1)[5]. Therefore, there is only one technique in this economy: $(a_{11}, l_1, a_{12}, l_1)$. The uniform nominal wage rate is denoted by w, the uniform profit rate by r, and the price of commodity $j = 1, 2$ by p_j .In both sectors, prices must

[4] Therefore "[…] the classical economists' idea of a wage 'advanced' from capital" (Sraffa, 1960, p. 11) must be abandoned.

[5] We can use each pair of methods in a technique, as Garegnani (1970).

cover wage costs (wl_i), the cost of capital used up ($p_j a_{1j}$) and profit paid to capitalists at the uniform rate on the value of advanced capital ($rp_j a_{1j}$). The consumption good (bread) is assumed to be the unit of measure, that is, the second commodity is the *numéraire*. Given all the above assumptions, this is the system of price equations:

$$\begin{gathered} p_1 = p_1 a_{11}(1+r) + wl_1 \\ p_2 = p_1 a_{12}(1+r) + wl_2 \\ w = p_2 b \\ p_2 = 1 \end{gathered} \tag{1}$$

Thus,

$$\begin{gathered} p_1 = p_1 a_{11}(1+r) + bl_1 \\ 1 = p_1 a_{12}(1+r) + bl_2 \end{gathered} \tag{2}$$

Where b is the real wage in terms of the second commodity. It is relevant to emphasize that p_1 is not "absolute" price of corn but the relative price of corn. This is a system of two equations and three unknowns (p_1, b and r).[6] To solve this system, assume that profit rate is the independent variable, as Sraffa (1960, §44) does. This assumption is compatible with the wage paid after the end of the production period. Thus, there is enough data to solve this system. To find p_1, first isolate b in the second equation of system (2) and then substitute the result into the first equation of system (2). After rearranging the first equation of that system, the relative price of corn[7] is obtained:

$$p_1 = \frac{l_1}{(l_1 a_{12} - l_2 a_{11})(1+r) + l_2} \tag{3}$$

On the other hand, by eliminating p_1 from the system of price equations, the solution for the real wage is obtained:

$$b = \frac{1 - a_{11}(1+r)}{l_2 + (l_1 a_{12} - l_2 a_{11})(1+r)} \tag{4}$$

This last equation is a function that relates a single value of b to any value of r. It is the wage curve. Both variables are negatively related to each other, both because of the negative sign of r in the numerator and because r is in the denominator: a fall in r increases the numerator and decreases the denominator of this function, assuming the other variables constant, and thus b increases. The real wage and the profit rate move in opposed directions. The vertical and horizontal intercepts of wage curve are given by the maximum real wage and maximum profit rate, respectively. These can be found from the function which gives rise to the wage curve. For $r = 0$, one obtains the maximum real wage (B):

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[6] If the capital goods were not all circulating ones, that is, if they were not completely used up during a single year, there would be fixed capital in the economy, so that prices equations would have to consider depreciation of capital. Thus, the equations for each commodity would be: $p_1 = p_1 a_{11}(d+r) + wl_1$ and $p_2 = p_1 a_{12}(d+r) + wl_2$, where d is rate of depreciation. Therefore, in the economy here analyzed, $d = 1$. When there is fixed capital, several results discussed in this chapter change. Samuelson (1962) assumes that $d = 1/n$. But this way is not correct because it violates the uniform profit rate condition as shown by Sraffa (1960) in chapter on fixed capital.

[7] From now on, p_1 refers to the price of corn relative to the price of bread. If this is not the case, it will be specified.

$$B = \frac{1 - a_{11}}{l_1 a_{12} + l_2(1 - a_{11})} = \frac{1}{l_2 + \left(\frac{a_{12}}{1 - a_{11}}\right) l_1} \tag{5}$$

The denominator of the last fraction in equation (5) refers to the amount of direct and indirect labor required to produce one unit of consumption good. Given the assumptions on net output and *numéraire*, B is the maximum real wage per unit of labor and measures net output per unit of labor. And for $b = 0$, one obtains the maximum profit rate (R):

$$R = \frac{1 - a_{11}}{a_{11}} \tag{6}$$

The maximum profit rate depends only on the physical surplus of corn – on the technical conditions in first sector –, because only corn is a basic good which is used as input in production of itself and bread. When $b > 0$, and $r < R$, the conditions in the sector producing bread affect the profit rate. The integrated industry of the consumption good (or integrated industry) – the integration of the capital goods and consumption goods sectors, whose methods give rise to a technique – ensures the replacement of corn consumed in production of itself and bread. So, the quantity of capital good X_1 produced by this industry replaces the proportion of capital good consumed in both production processes. In this way:

$$X_1 = a_{11}X_1 + a_{12}X_2 \tag{7}$$

Assuming consumption good is normalized, $X_2 = 1$, and modifying equation (7), one obtains:

$$X_1 = \frac{a_{12}}{1 - a_{11}} \tag{7'}$$

In this way, X_1 turns out to be "[…] the proportion of the capital good consumed in the production of the proportion of the capital good necessary for producing the consumption good" (Lazzarini, 2011, p. 44). Thus, the net physical output per worker of the integrated industry, q, is given by:

$$q = \frac{X_2}{L} = \frac{X_2}{l_1 X_1 + l_2 X_1} = \frac{1}{l_1 X_1 + l_2} = \frac{1}{l_2 + \left(\frac{a_{12}}{1 - a_{11}}\right) l_1} \tag{8}$$

As can be seen, equation (8') is equal to the maximum real wage (equation 5), which represents the maximum physical quantity of consumption good received by workers when $r = 0$. Thus, the maximum real wage is equal to productivity of the integrated industry. Although the inverse relationship between the real wage and the rate of profit has already been shown by equation (4), the derivative of b with respect to r in the equation below shows how this trade-off is unambiguously downward:

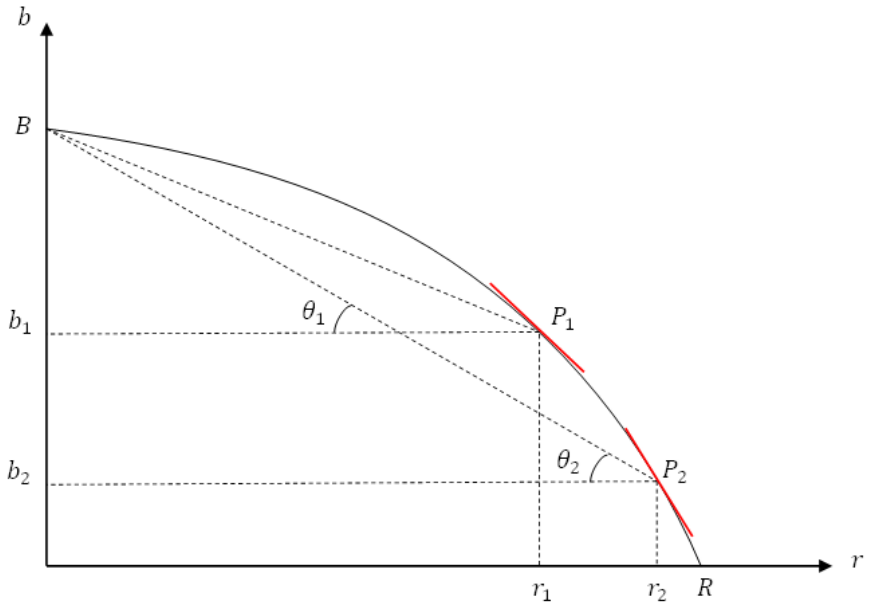
$$\frac{db(r)}{dr} = -\frac{l_1 a_{12}}{[l_2 + (l_1 a_{12} - l_2 a_{11})(1 + r)]^2} \tag{9}$$

The derivative (9) measures how much the real wage changes when the profit rate changes. The specific shape of the real wage-profit curve, whether concave, convex or linear, depends on the technology and *numéraire*. Regarding the first one, it depends on the following relationships:

$$l_1 a_{12} - l_2 a_{11} \lessgtr 0 \text{ and } l_1 a_{12} \lessgtr l_2 a_{11} \text{ as } \frac{a_{11}}{l_1} \gtrless \frac{a_{12}}{l_2}$$

When the first sector is physically more capital-intensive than the second sector, i.e., $(a_{11}/l_1) > (a_{12}/l_2)$, the term $l_1 a_{12} - l_2 a_{11}$ of equation (4) is less than zero and the wage curve will be concave, as can be seen in Figure 1 below. In this case, as the rate of profit increases and the real wage decreases, the relative price of the capital good (corn) increases. This is because when the profit rate increases (decreases), the denominator of the relative price-profit equation (3) decreases (increases) and the relative price of corn increases (decreases), since the sector that produces corn requires more capital and is relatively more affected by the increase (decrease) in the profit rate. Therefore, p_1 is a positive function of r.

Figure 1 – The (concave) wage-profit curve



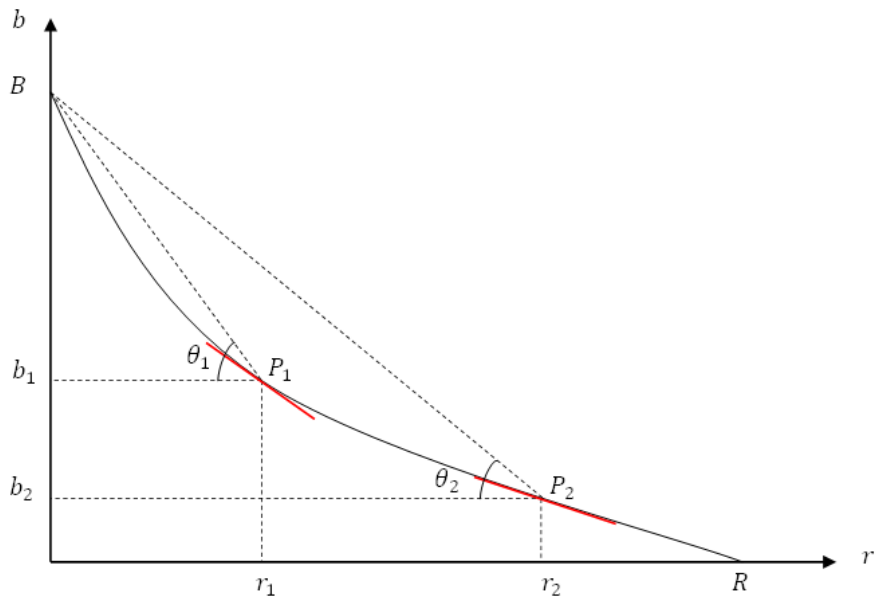
Source: Own elaboration.
The red lines are tangent lines. As the profit rate increases, the tangent line becomes more vertically inclined. The tangent of angle θ_2 is greater than the tangent of angle θ_1.

When that curve is concave, the fall (rise) in real wage will accelerate as profit rate increases (decreases). Thus, the value of capital per worker increases as the profit rate increases, since the relative price of corn increases. The value of capital per worker is, geometrically, the trigonometric tangent of the angle formed by the straight line connecting the output per worker of the technique (the maximum real wage, B) to some point on the wage-profit curve corresponding to some real wage at that point, given the profit rate (r). Consider, for example, the angle θ_1, formed by $b_1 P_1 B$, in Figure 1 above. As the profit rate increases from r_1 to r_2, the slope of the curve increases in magnitude (the

curve becomes more negatively steep or negatively vertical), from point P_1 to P_2, and so does the value of capital per worker.

On the other hand, when the second sector is physically more capital intensive than the first sector, i.e., $(a_{11}/l_1) < (a_{12}/l_2)$, the term $l_1a_{12} - l_2a_{11}$ of equation (4) is greater than zero and the wage curve will be convex, as can be seen in Figure 2 below. In this case, as the rate of profit increases and the real wage decreases, the relative price of the capital good (corn) decreases. This is because when the profit rate increases (decreases), the denominator of the relative price-profit equation (3) increases (decreases) and the relative price of corn decreases (increases), since the sector that produces bread requires more capital and is relatively more affected by the increase (decrease) in the profit rate. Therefore, p_1 is a negative function of r.

Figure 2 - The (convex) wage-profit curve

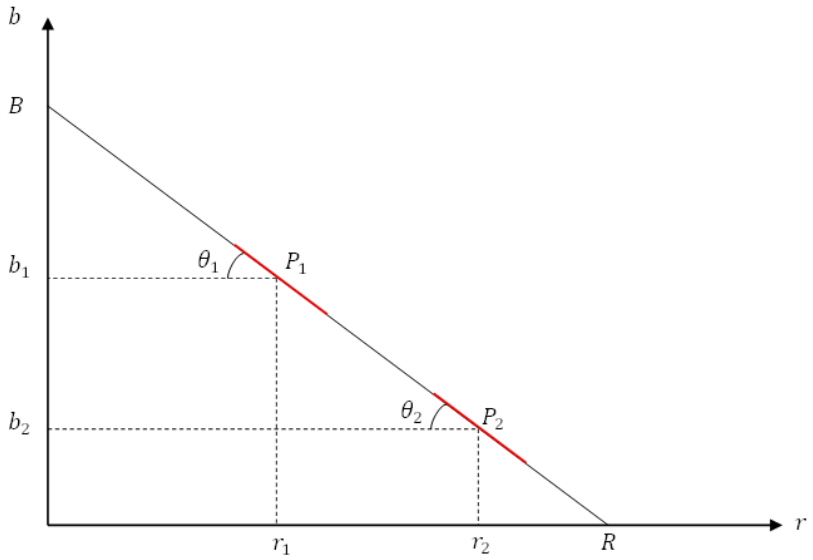


Source: Own elaboration.
The red lines are tangent lines. As the profit rate increases, the tangent line becomes less negatively vertical. The tangent of angle θ_2 is less than the tangent of angle θ_1.

When the wage curve is convex, the fall (rise) in real wage will slow down as profit rate increases (decreases). Thus, the value of capital per worker decreases as the profit rate increases, since the relative price of corn decreases. In Figure 2, as the profit rate increases from r_1 to r_2, the slope of the curve decreases in magnitude (the curve becomes less negatively steep or negatively vertical), from point P_1 to P_2, and so does the value of capital per worker.

Whereas, when capital-labor ratios in both sectors are identical, i.e., $(a_{11}/l_1) = (a_{12}/l_2)$, the term $l_1 a_{12} - l_2 a_{11}$ of equation (4) is equal to zero and the wage curve will be linear[8], as can be seen in Figure 3 below. In this case, the relative price of corn is proportional to the amount of labor directly required to produce corn and bread – it is independent of profit rate –, because the first term in parentheses in the denominator of equation (3) is equal to zero. The value of capital per worker, which is measured by the slope of the wage curve, remains constant. In Figure 3 below, as the rate of profit increases from r_1 to r_2, the slope of the curve remains constant from point P_1 to P_2, and the same occurs with the value of capital per worker.

Figure 3 – The (linear) wage-profit curve



Source: Own elaboration.
The red lines are tangent lines. As the profit rate increases, the tangent line remains constant. The tangent of angle θ_2 is equal to the tangent of angle θ_1.

Note that when the wage curve is linear, capital per worker is constant, since only in this case the relative prices are independent of distribution. Only in this case, the first derivative of b with respect to r is equal to zero, which means that the slope does not change along the linear wage curve. In this case, the first term in parentheses in the denominator of equation (4) is equal to zero and, therefore, the wage-profit curve is given by this equation:

$$b = \frac{1 - a_{11}}{l_2} - \frac{a_{11}}{l_2} r \qquad (10)$$

[8] In the economy here analyzed, the wage curve may be concave, convex or linear (special case). But if in an economy there are more than two basic goods, that curve may have several inflections, whose amount is equal to the number of basic goods. That is, its segments may alternate between concave and convex ones (Petri, 2004, p. 208).

In this case, the first term after the equals sign in equation (10) is equal to the maximum real wage: $(1 - a_{11})/l_2$. Only in this situation, where relative prices are independent of distribution, will the slope of the wage-profit curve be given by the *physical* capital-labor ratio ($-a_{11}/l_2$) and will be constant, given the quantities of corn required to produce one unit of corn and the direct labor required to make one unit of bread. Thus, when the wage-profit curve is not linear, the value of capital per worker changes as the income distribution changes, even if the capital-labor ratio in physical terms remains constant.[9]

Moreover, the shape of wage curve also depends on the *numéraire*, and not just on technical coefficients (including labor ones) – although it is unambiguously downward sloping. If commodities are measured in terms of corn instead of bread, that is, $p_1 = 1$, that curve will be convex when the first sector is physically more capital intensive than the second sector, $\frac{a_{11}}{l_1} > \frac{a_{12}}{l_2}$. It will be concave when the second sector is physically more capital intensive than the first sector, $\frac{a_{12}}{l_2} > \frac{a_{11}}{l_1}$. But independently of the *numéraire* chosen, when capital-labor ratios in both sectors are identical, the wage curve is linear.

2.2 CHOICE OF TECHNIQUES

Now assume that there are two different available techniques to produce net output in this capitalist economy – net output consisting only of a certain amount of consumption good (bread). Each technique encompasses two methods of production for each sector. In each technique, capital good is produced by itself and labor through a single method specific to it and bread is produced by labor and capital good. The capital goods employed by different techniques are distinct from one another. These distinctions can manifest in two forms. First, the quantities of the same capital good required by each technique to produce one unit of the consumption good may differ. For example, one technique might require a specific quantity of a capital good (e.g., corn), while another requires a different quantity of the same capital good (e.g., corn). This leads to different capital-labor ratios between the techniques.[10]

Table 1 – Technical coefficients

	Method to produce 1	Method to produce 2
Technique α	$a_{11}^{\alpha}, l_1^{\alpha}$	$a_{12}^{\alpha}, l_2^{\alpha}$
Technique β	$a_{11}^{\beta}, l_1^{\beta}$	$a_{12}^{\beta}, l_2^{\beta}$

Source: Own elaboration.

[9] In the general case, when $(a_{11}/l_1) \neq (a_{12}/l_2)$, relative prices change as the distribution changes. The relationship between relative prices and profit rate is monotonic: the sign of the first derivative of the relative price of corn with respect to the profit rate remains the same. There are only the prices of corn and bread. When the capital goods sector is the most (least) capital goods intensive, its relative price is a positive (negative) function of the profit rate. The relationship is positive, negative, or nonexistent – therefore, monotonic. Only when there is more than one capital good does the relationship cease to be monotonic.

[10] For classical economists and Marx, the definition of different capital goods refers to different quantities.

Second, the techniques may require entirely different kinds of capital goods. To illustrate, one technique uses corn as a capital good. In this system, corn is produced using itself and labor via a specific method, and is then used with labor to produce bread. A different technique uses rye as its capital good, which is also produced using itself and labor, and is then used to produce bread. In this simplified way, there are only two techniques. The commodities produced in each are basic, and the capital goods are specific to each technique. Strictly speaking, when techniques use different kinds of capital goods, the resulting consumption goods may also be considered distinct – a point noted by D'Ippolito and Potestio (1993). However, the fundamental point remains that the capital good used by each technique is different, whether in magnitude or in kind.

Because of those assumptions, one must establish the criterion adopted by the capitalist to choose the method between available ones to produce consumption good. Following Sraffa (1960) and several other authors, the criterion adopted is profit maximization, or symmetrically, cost minimization. That choice depends on the production costs of consumption good, which in turn depends on the technique adopted. But each technique sets up a different set of relative prices and a different real wage for the given profit rate. Thus, to compare the supply prices of each method it is required to evaluate both techniques at both sets of prices. For each technique, the supply prices are:

Technique (α)

$$\begin{aligned} p_1^\alpha &= p_1^\alpha a_{11}^\alpha (1+r) + w^\alpha l_1^\alpha \\ p_2^\alpha &= 1 = p_1^\alpha a_{12}^\alpha (1+r) + w^\alpha l_2^\alpha \\ p_{1(\alpha)}^\beta &= p_{1(\alpha)}^\beta a_{11}^\beta (1+r) + w^\alpha l_1^\beta \\ p_{2(\alpha)}^\beta &= p_{1(\alpha)}^\beta a_{12}^\beta (1+r) + w^\alpha l_2^\beta \\ b^\alpha &= \frac{w^\alpha}{p_2^\alpha} \end{aligned} \tag{11}$$

Technique (β)

$$\begin{aligned} p_1^\beta &= p_1^\beta a_{11}^\beta (1+r) + w^\beta l_1^\beta \\ p_2^\beta &= 1 = p_1^\beta a_{12}^\beta (1+r) + w^\beta l_2^\beta \\ p_{1(\beta)}^\alpha &= p_{1(\beta)}^\alpha a_{11}^\alpha (1+r) + w^\beta l_1^\alpha \\ p_{2(\beta)}^\alpha &= p_{1(\beta)}^\alpha a_{12}^\alpha (1+r) + w^\beta l_2^\alpha \\ b^\beta &= \frac{w^\beta}{p_2^\beta} \end{aligned} \tag{12}$$

Where $p_{j(\beta)}^\alpha$, $j = 1,2$, is the supply price of commodity j produced with method α and with cost determined by prices and real wage of method β. While $p_{j(\alpha)}^\beta$, $j = 1,2$, is the supply price of commodity j produced with method β and with cost determined by prices and real wage of method α. And real wage of techniques α and β are b^α and b^β, respectively. One must assume the analysis

is framed in terms of a given technique's price; otherwise, multiple complex relationships will emerge, rendering technique comparisons unfeasible. Therefore, capitalist will adopt the method of production yielding the lowest supply price for the given commodity at the given prices, profit rate and real wage and so that he or she obtains extra profits.

If technique α is known initially, then, to compare both techniques, prices and wages are denominated in terms of technique α. The comparison must be made between p_j^{α} and $p_{j(\alpha)}^{\beta}$. Thus, the wage curve for technique α is:

$$b^{\alpha} = \frac{1 - a_{11}^{\alpha}(1+r)}{l_2^{\alpha} + (l_1^{\alpha}a_{12}^{\alpha} - l_2^{\alpha}a_{11}^{\alpha})(1+r)} \tag{13}$$

And the relative price of corn produced in technique α with prices and distribution determined in this technique is:

$$p_1^{\alpha} = \frac{l_1^{\alpha}}{(l_1^{\alpha}a_{12}^{\alpha} - l_2^{\alpha}a_{11}^{\alpha})(1+r) + l_2^{\alpha}} \tag{14}$$

So, using the real wage determined by technique α, the supply prices $p_{1(\alpha)}^{\beta}$ and $p_{2(\alpha)}^{\beta}$ in equations (11) are determined. Given this, the relevant comparison is with the *numéraire*, because $p_2^{\alpha} = 1$. Thus, if $p_{2(\alpha)}^{\beta} < 1$, the technique β is cheaper and, therefore, will be adopted. Otherwise, if $p_{2(\alpha)}^{\beta} > 1$, the technique β is the most expensive and therefore will not be adopted.

Otherwise, if technique β is initially known one, the comparison must be made between p_j^{β} and $p_{j(\beta)}^{\alpha}$. So, prices and wages are denominated in terms of technique β. The wage curve for technique β is:

$$b^{\beta} = \frac{1 - a_{11}^{\beta}(1+r)}{l_2^{\beta} + (l_1^{\beta}a_{12}^{\beta} - l_2^{\beta}a_{11}^{\beta})(1+r)} \tag{15}$$

And the relative price of corn produced in technique β with prices and distribution determined in this technique is:

$$p_1^{\beta} = \frac{l_1^{\beta}}{\left(l_1^{\beta}a_{12}^{\beta} - l_2^{\beta}a_{11}^{\beta}\right)(1+r) + l_2^{\beta}} \tag{16}$$

Thus, using the real wage determined by technique β, the supply prices $p_{1(\beta)}^{\alpha}$ and $p_{2(\beta)}^{\alpha}$ in equations (12) are determined. Given this, the relevant comparison is with the *numéraire*, because $p_2^{\beta} = 1$. Therefore, if $p_{2(\beta)}^{\alpha} < 1$, technique α is cheaper and, therefore, will be adopted. Otherwise, if $p_{2(\beta)}^{\alpha} > 1$, technique α is more expensive and, therefore, will not be adopted.

The shape of the real wage-profit rate curve for each technique depends on the same elements that determine the shape of the real wage-profit rate curve in the economy where there is only one available method for producing both corn and bread. That is, the shape of that curve will be concave, convex or linear depending on the *numéraire* and technical coefficients (including labor ones).

Regardless of the shape, it is still unambiguously downward sloping. Therefore, given two methods of production, the choice of technique depends on whether the capitalist initially has method α or β. Based on what has been presented regarding the choice of techniques, it is only possible to evaluate the processes of technique β under α's prices (or vice versa), if they contain the same capital goods. That is: the same capital good, but in different quantities in each technique.

Nevertheless, a problem arises if the capital goods differ between the techniques(for example, technique α uses corn and technique β uses rye): the methods $(I\beta)$ and $(II\beta)$ cannot be evaluated at technique α's price, since those methods contain rye and labor (as input and/or output) for which latter technique has no corresponding price. And if the technique β is initially considered, the same problem emerges: the methods $(I\alpha)$ and $(II\alpha)$ cannot be evaluated at technique β's price, because those methods contain corn and labor (as input and/or output) for which latter technique has no corresponding price. The problem is qualitative one: it is not possible to determine the value of $p^{\alpha}_{j(\beta)}$ and neither $p^{\beta}_{j(\alpha)}$. But it is possible to determine the ratios between the techniques' prices, because both techniques are structurally identical. Thus:

$$p_1^{\alpha}: p_2^{\alpha}: p_{1(\alpha)}^{\beta}: p_{2(\alpha)}^{\beta}: b^{\alpha} = p_1^{\beta}: p_2^{\beta}: p_{1(\beta)}^{\alpha}: p_{2(\beta)}^{\alpha}: b^{\beta} \tag{17}$$

And one obtains:

$$\frac{b^{\alpha}}{b^{\beta}} = \frac{p_2^{\alpha}}{p_{2(\beta)}^{\alpha}}$$
$$\frac{b^{\alpha}}{b^{\beta}} = \frac{p_2^{\beta}}{p_{2(\alpha)}^{\beta}} \tag{18}$$

If these ratios are equal both techniques will be equally profitable at both techniques' real wage and prices, and they can coexist. But technique α will be the cheapest for producing consumption good if $b^{\alpha} > b^{\beta}$, then $p_2^{\beta} > p_{2(\alpha)}^{\beta}$. Otherwise, if $b^{\alpha} < b^{\beta}$, then $p_2^{\beta} < p_{2(\alpha)}^{\beta}$ and the technique β is the cheapest. Therefore, the cost-minimizing technique is that which gives the highest real wage for the given profit rate. The choice of technique can be inferred from wage curve in terms of any *numeráire*.

3 Samuelson's (1960) surrogate production function and Garegnani's (1970) critique

Samuelson (1962) considers his surrogate production function "[…] as a dramatic model to show that mere *physical* heterogeneity need not lead to qualitatively new behavior patterns" (Samuelson, 1962, p. 203, author's italics). According to the author, his model consists of a variety of capital goods which produces with labor only one kind of homogeneous final output. Capital goods are specific to each productive process and thus each capital good of each technique is a different one. Heterogeneous physical capitals measured in value terms are assumed in Samuelson's (1962) model – differently of Clark-Ramsey parable, according to which supposes "homogeneous physical

capital jelly", in view of this author. Capital good is produced by itself and labor while consumption good is produced by capital good and labor. Only one kind of consumption good is produced[11].

His model assumes fixed proportions of coefficients, constant returns to scale and perfect competition for each technique. Nevertheless, these are the most relevant assumptions made by Samuelson (1962, p. 196-197):

> […] I assume that any one capital good, call it alpha, looks entirely different from a second beta capital good. […] I shall here assume that the same proportion of inputs is used in the consumption good [sic] and alpha goods [sic] industries […]. The present model, however, does not require any of beta to help in producing alpha, nor vice versa […].

This means that capital goods are entirely different among them, in the sense that a capital good of one technique is different from another capital good of another technique. For a given technique, the ratio of capital good to labor is the same in sector producing capital good and sector producing consumption good. And each capital good of one technique does not need any other capital good of another technique to be produced and vice versa. Thus, within each technique, the capital-labor ratios in the sectors producing capital goods and consumption goods are the same. However, these ratios differ between techniques. This is a direct consequence of Samuelson's (1962) assumption that each technique uses a distinct capital good. Moreover, these different capital goods are naturally occurring and form part of the initial capital endowment. Thus, the wage curve for each technique, α and β, is given by equation (10), and illustrated in Figure 4 below.

In addition, he considers fixed capital, instead of circulating capital, whose mortality rate is independent of age: the physical depreciation of capital is directly proportional to the physical capital good (Samuelson, 1962, p. 197). According to Samuelson (1962), considering that these assumptions are held for several techniques, all wage curves would be straight lines, and it would be possible to find the family of techniques which gives rise to a wage-profit frontier.[12]

The wage-profit frontier, in turn, would become an envelope whose slope measures the capital-labor ratio of the economy. In such a way, techniques and capital-labor ratio change as a continuous function of profit rate and only one technique is adopted for each feasible profit rate. Thus, it would supposedly be possible to find a functional relationship between net output per worker and distributive variables and techniques could be ordered according to net output per worker or to capital

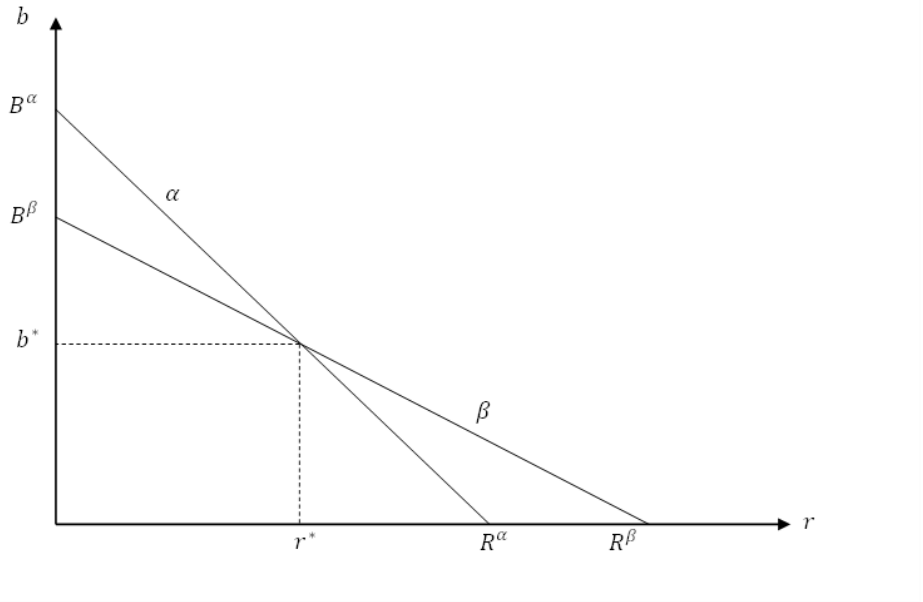
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[11] Samuelson (1962) assumes this in section two of his work – "[…] society produces only on kind of homogeneous final output […]" (p. 194) – and section three where his special model is presented – "[…] every stationary state produces exactly the same output related to the size of total labor employed […]" (p. 198).

[12] "[…] if all the wage-curves are straight lines or convex ones the envelope will be convex, whereas the latter will be concave or straight when the wage-curves are concave" (Lazzarini, 2011, p. 64). Thus, if the envelope of the wage curve is convex, the value of capital per worker varies inversely with respect to the rate of profit, as marginalist theory assumes.

per worker. Since criterion adopted by capitalists for choosing of techniques is cost minimization, as profit rate (real wage) increases capital intensity falls (rises).

Figure 4 – Samuelson's model



Source: Own elaboration.

In another words, there is always an inverse relationship between prices production factors[13] and the intensity in which they are used in the methods adopted. This relationship could be summarized by a continuous and differentiable production function or surrogate production function, according to which changes in net output per worker would correspond to changes in capital-labor ratio, which changes inversely to the profit rate. Or, in the same sense, net output per worker would be directly proportional to the real wage.

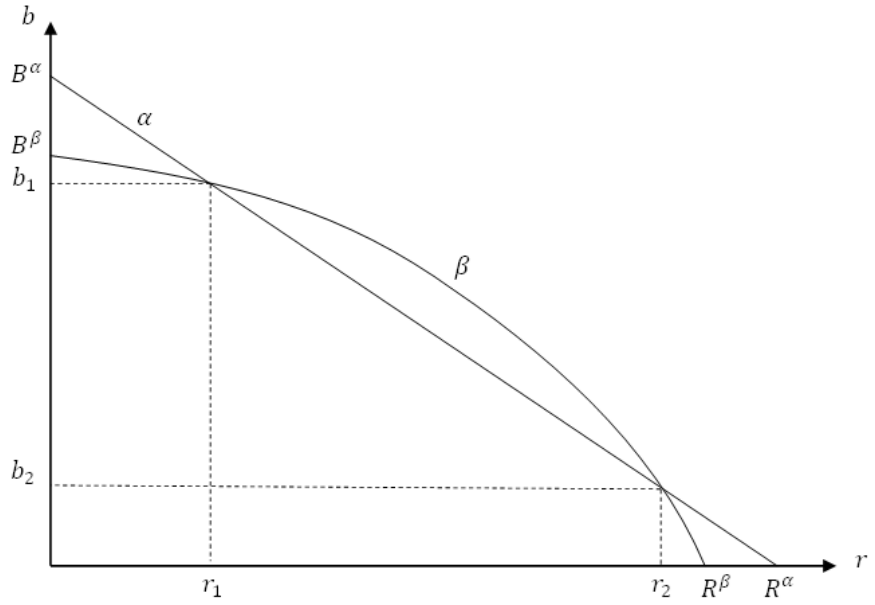
However, Garegnani (1970) exposes several problems with Samuelson's (1962) model. First, since the envelope of wage curve can be concave to the origin, in parts or throughout, as profit rate increases, the capital-labor ratio also increases. In that case, the price of capital good is a positive function of profit rate and, thus, the value of this ratio goes in opposite direction to that established by the surrogate production function. Consider the following example, which is illustrated in Figure 5 below. Suppose that in technique α capital-labor ratios in both sectors are identical so that its wage curve is given by equation (10). And in technique β, assume that the first sector is "physically more capital intensive" than the second sector, so the wage curve is concave with respect to the origin, its general shape being given by equation (4), and the value of capital-labor ratio is an increasing function of the profit rate.

[13] The term factor of production corresponds to the marginalist idea of a production process as one-way avenue (Sraffa, 1960, p. 111; Fratini, 2024, p. 3).

Further, assumes that bread is the *numeráire* – it must be the same in determining both wage curves – and the vertical and horizontal intercepts of the wage curve of the α technique are the largest, that is, this technique has the highest maximum real wage and the highest maximum profit rate. So, under these assumptions, technique α is "more capital intensive". The α technique has a higher value of capital per worker than the β technique. The slope of the wage-profit curve generated by the α technique is greater than that of the β technique.

In this case both wage curves can cross twice in the positive orthant. At profits rates r_1 and r_2 switch of techniques occurs, i.e., a change in technique chosen by the capitalist occurs as income distribution changes. In the economy under scrutiny, when the profit rate is between 0 and r_1, that is, $0 < r < r_1$, technique α is chosen. When $r = r_1$ or $r = r_2$ both techniques are equally profitable (cost minimizing), so that at given r and in terms of any *numéraire* both real wages and relative prices (denominated in terms of technique α) of each technique are equal, and can coexist. But when $r_1 < r < r_2$, the switch of techniques occurs and technique β is the chosen one. And when the profit rate increases beyond r_2, technique α is once again the chosen one. In the last case the phenomenon reswitching of technique occurs: the same technique is adopted in different levels of the profit rate.

Figure 5 – Garegnani's critique



Source: Own elaboration.

Further, the phenomenon reverse capital deepening occurs also. When $0 < r < r_1$ technique "more capital intensive" ("less labor intensive") is chosen and when $r_1 < r < r_2$ technique "less capital intensive" ("more labor intensive") is chosen. This movement occurs according to the marginalist approach: as the rate of profit increases, the technique adopted is the one that is less capital intensive. But when $r_2 < r < R^\alpha$, the technique α which is "more capital intensive" is adopted even

with the profit rate increasing. In this case, the reverse capital deepening happens and there is a positive relationship between the profit rate and the value of capital per worker, since technique α's value of capital per worker is higher than technique β's one.[14] This case "[…] will happen if the two wage curves had already crossed (once, or an odd number of times) at lower values of the rate of interest […]" (Petri, 2004, p. 224).

The value of capital per worker not only varies when there is more than one technique, and one of these techniques uses different capital-labor ratios between sectors, but it also varies in the opposite direction to what is assumed by marginalist theory. When technique α is chosen, the value of capital per worker remains constant despite changes in the profit rate. However, when technique β is chosen, the slope of the curve increases as the profit rate rises, and consequently, so does the value of capital per worker. Furthermore, when technique α, which has a higher value of capital per worker compared to technique β (i.e., the slope of α is steeper than that of β), is once again chosen even with the higher profit rate.

Therefore, the surrogate production function "[…] could not be a production function […]" (Garegnani, 1970, p. 415). Moreover, regarding that equilibrium in the Clark-Ramsey parable needs that the marginal product of capital does not rise when capital-labor ratio rises, it would face a problem even in the case where the envelope is a straight-line, in parts or throughout. This is because, in case where outer envelope of all wage curves is a straight-line, capital-labor ratio does not change, as do the "marginal products", which contradicts the idea of "marginal changes" that occur as the proportions of the factors change.

Furthermore, there would not be an equivalence between net output per worker of Clark-Ramsey parable and of surrogate production function in general cases where the wage-curves are convex or concave to the origin – in the former case the latter function underestimates net output per workers, while the latter case overestimates at all levels of the profit rate. Only when all wage curves are straight lines will that equivalence occur, as well as the inverse relationship between capital-labor ratio and the profit rate. Consequently, that function "[…] exists, *if and only if* all wage-curves are straight lines" (Garegnani, 1970, p. 415, author's italics). Given in Samuelson's (1962) model the proportions between capital and labor in both sectors are the same, which is a very particular case, this means there is one consumption good which is produced by itself and labor. Capital is homogenous instead heterogeneous, like "Clark-Ramsey parable. Hence, surrogate production function is the aggregate production function (Garegnani, 1970, p. 415). Because of that Samuelson (1962) can avoid capital demand-side critique.

[14] Here the analysis is in terms of reverse capital deepening and not reverse labor deepening which mean that "lowering the real wage may lead to a decrease in total (direct and indirect) employment". In this way, it would be easier analyzing under this last term because labor intensity does not depend on prices and can be measured in physical terms.

Besides, strictly speaking, in the economy there cannot be fixed capital in the economy also. Since Samuelson (1962) assumes that physical depreciation is independent of age, "the problems specific to *fixed* capital, which are problems of joint production […]" (Garegnani, 1970, p. 403), are evaded. But, when an appropriate approach to fixed capital is considered, rather than the one considered by Samuelson (1962), the proportions of means of production to labor required to produce commodities are different and, therefore, the wage-curves are not linear (Steedman, 1979, 2019).

4 Baldone (1984), "non-switching" and further considerations on the Samuelson's (1962) model

In addition to Garegnani's (1970) criticism of Samuelson (1962), there is Baldone's (1984) criticism which, when considering the case in which capital intensities are the same in both sectors, shows that the conditions for the validity of the surrogate production function are more restrictive. Consider the economy depicted in section 2, in which there are those two techniques (α and β), with the consumption good being the sole commodity common to both techniques, and the assumptions made by Samuelson (1962). The wage curves of each technique are given by:

$$b^{\alpha} = \frac{1 - a_{11}^{\alpha}}{l_2^{\alpha}} - \left(\frac{a_{11}^{\alpha}}{l_2^{\alpha}}\right) r \tag{19}$$

$$b^{\beta} = \frac{1 - a_{11}^{\beta}}{l_2^{\beta}} - \left(\frac{a_{11}^{\beta}}{l_2^{\beta}}\right) r \tag{20}$$

At switch point, both techniques are cost minimizing and equally profitable at the given level of profit rate. Therefore, their real wages (and relative prices) would be the same. Now, given that there is a common activity between the techniques and the capital-labor ratios in both sectors are the same within each technique, the capital-labor ratios between techniques α and β are also the same.

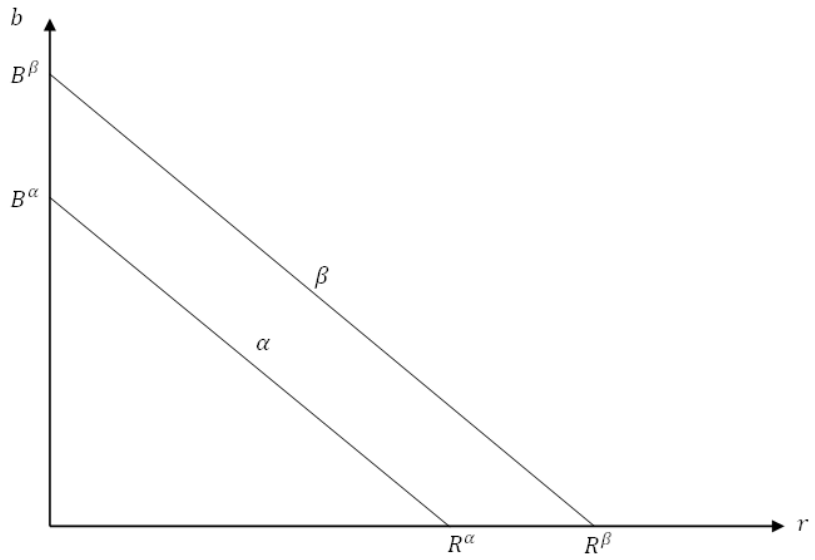
In other words, the capital-labor ratios in different techniques are the same for producing the same consumption good (e.g., bread); that is, the capital good in one technique (e.g., corn) is the same as in the alternative technique, but the magnitudes of capital and labor are different, because otherwise the techniques would be the same. Thus, in the economy considered, the proportions between capital and labor in both sectors and to both techniques are equal: $a_{11}^{\alpha}/l_1^{\alpha} = a_{12}^{\alpha}/l_2^{\alpha} = a_{11}^{\beta}/l_1^{\beta} = a_{12}^{\beta}/l_2^{\beta}$. In consequence, if the coefficients of capital and labor in technique α are greater than the coefficients in technique β, $a_{11}^{\alpha} > a_{11}^{\beta}$ (which implies $l_1^{\alpha} > l_1^{\beta}$ to maintain the same capital-labor ratios between techniques), so necessarily the second technique dominates the first one. Figure 6 below illustrates this case.

In contrast, if the coefficients of the first technique are smaller than those in the second technique, $a_{11}^{\alpha} < a_{11}^{\beta}$ (which implies $l_1^{\alpha} < l_1^{\beta}$ to maintain the same capital-labor ratios between techniques), so necessarily technique α is superior to the technique β. Therefore, technique β has both maximum real wage and maximum profit rate greater or smaller than those of technique α. This

can be seen from equations (19) and (20) looking at the vertical intercept (first term after the equal sign) and horizontal intercept (which is obtained from $b^{\alpha} = b^{\beta} = 0$).

Consequently, the wage curves are parallel. It follows that no switching point is possible between techniques that share the same "organic composition of capital". For the surrogate production function to exist, not only must all methods within a technique share the same capital-labor proportions, but these proportions must also differ between techniques. Furthermore, there cannot be any common production methods shared across techniques. The first and third conditions require that the wage curves be straight. The second condition, however, requires that the wage curves intersect, since with equal slopes, given by the same capital-labor ratio, they would not intersect and there would be no switch of techniques. This reveals how useful the assumptions made by Samuelson (1962) are in legitimizing marginalist theory through his model and, at the same time, how restrictive they are.

Figure 6 – "Non-switching" of techniques



Source: Own elaboration.

Even Garegnani (1970) does not emphasize this. Despite that, according to this author, it is enough that there is a common production method between two different techniques for at least one of the two wage-profit curves to be non-linear and reswitching of techniques can happen. That is, the existence of a productive technique which combines the methods (a_{12}^{α}, l_2^{α}) and (a_{11}^{β}, l_1^{β}) or which combines the methods (a_{11}^{α}, l_1^{α}) and (a_{12}^{β}, l_2^{β}) implies non-linear wage curves.

Samuelson (1962) arbitrarily assumes that capital goods of one technique are different from those of the other techniques and vice versa. Then, when technical changes (e.g. from technique α to technique β) all coefficients must change at once. Furthermore, this assumption runs counter to the

choice of techniques made by the capitalist in the real world: there is no such limitation on the use of capital goods employed in different techniques. Thus, if the wage-profit curves are very nonlinear, one falls into Garegnani's (1970) critique. If they are not very nonlinear, one falls into Baldone's (1984) critique.

Interestingly, on 16 January 1946, Sraffa had anticipated the surrogate production function and even the critique of Baldone (1984):

> The Irony of it is, that if the 'Labor [sic] Theory of Value' applied exactly throughout, then, and only the, would the 'marginal product of capital' theory work! It would require that all products had the same org[anic] comp[osition]; and that at each value of r each comm[odity] had an 'alternative method', and that the relations within each pai should be the same (i.e. that marg[inal] prod[uct]s should be the same; + also the elasticities should be the same); so that, even when the System is switched, and another Org. Comp. came into being, it should be the same for all products. Obviously this would be equivalent to having only one means-product (wheat). Then, commodities would always be exchanged at their Values; and their relative Values would not change, even when productivity of labor increased (Sraffa D3/12/16, p. 34; author's underlining).

In addition, even assuming doubtful Samuelson's (1962) assumption about fixed capital, this author fails to explain how full employment is maintained with fixed capital when a change in distribution alters the chosen technique. In the presence of fixed capital, gross investment must replace circulating capital plus new capital goods needed to replace the wear and tear of fixed capital – Samuelson (1962, p. 197) takes this into account. When capital goods are utilized for more than one production process, in each of this process only a part of the entire capital stock is used up, so that a part of the workers employed in the economy is made "free". Thus, new capital to be demanded by the capitalist must replace worn out capital and maintain full employment – according to Garegnani (1978), investment demand function is the demand function for capital as a stock. This just happens if the technique is cost minimizing.

However, if interest rate is decreased, capitalists are induced, according to marginalist theory, to adopt techniques which require different capital goods of greater value per unit of labor, so that those "freed" workers are not absorbed by the new capital which employs only a fraction of the fully employed labor supply. Unemployment rises. After that change in distribution and techniques, capitalists using fixed capital which was not completely worn out continue employing labor force corresponding to that capital and are content to earn residual quasi-rents (Petri, 2021, p. 552).

The direct substitution mechanism will continue to operate as long as capital endowment consists of fixed capital. Consequently, the economy will continue to be below full employment. The indirect substitution mechanism, however, does not work. Full employment, which would be guaranteed by this mechanism, is not possible in Samuelson's (1962) surrogate production function. This is because the relative prices are assumed to be independent of income distribution – since the proportion of capital goods to labor is the same in both sectors producing capital goods and

consumption good. Hence, this author assumes only one consumption good because of this assumption. Moreover, even if there were several consumer goods, if the entire economy has the same capital-labor ratio, the relative prices of the most labor-intensive sectors will not fall, because all sectors have the same intensity. The indirect substitution mechanism still does not work.

The substitution mechanism required by Samuelson's (1962) model is prevented from providing the properties necessary for the surrogate production function to be derived. Therefore, to avoid unemployment caused by changes in techniques, which are caused by changes in income distribution – no matter how small it is – this function implicitly assumes that all production methods throughout the economy must change, that is, when there is a change of techniques, all capital and labor coefficients change at the same time. Surrogate production function relies on the extremely restrictive assumption that there is only circulating capital, since, in this case, the entire capital stock is renewed when there is a change in real wage, and full employment is reached directly in the new technique adopted.

5 Capital supply-side critique and the specificity of Samuelson's (1962) model

For the marginalist economy in its traditional version, the existence and uniqueness of long-period general equilibrium, where the profit rate is uniform, depends on the condition that set of capital goods demanded by the enterprises are equal to the given value of capital endowment of the economy. And for the notion of aggregate production function, it is necessary that each single level of output corresponds to a given quantity of factors of production. In the real economy, where capital goods are heterogeneous, it is necessary to measure capital endowment in value terms at a chosen *numéraire*. However, the value of the existing capital endowment ends up depending on income distribution, since the relative prices of capital goods depend on distribution between the profit rate and wages when the proportions between capital and labor required by the goods are different.

This would not be so if there was only one type of capital in the economy, because the value of the capital endowment would be equal to the physical quantity of the single capital good. Or if the capital intensity in all sectors of production was uniform, since relative prices would be independent on income distribution. Or if the *numéraire* was "[…] a composite commodity with the same composition as the existing capital stock […]" (Petri, 2004, p. 94). The first two situations are unrealistic and the third is arbitrary. Therefore, the value of capital endowment changes as income distribution changes even if nothing has happened to the capital stock. Consequently, there is no uniqueness of the relation between the output and the given quantity of factors of production, that is, not only that endowment is indetermined as well as production function.[15]

[15] Garegnani (1970, p. 425) is deliberately partially overlooking the problem of the neoclassical concept of endowment of capital.

The indeterminateness of the capital endowments problem has blunt implications on the marginalist explanation of demand of factors of production, specifically the demand curve for labor. This theory needs a given value of capital endowment to derive a labor demand curve negatively sloped which expresses an inverse relationship between demand for labor and real wage. But, since that endowment is not determined it is impossible "[…] *start* to discuss the shape of the long-period labor [sic] demand curve because one does not possess sufficient data for its determination" (Petri, 2004, p. 297, author's italics). Since the measurement of quantity of capital is not independent of prices and income distribution, capital endowment, capital supply curve and production function are all indetermined. There is not a single productive factor called "capital" capable of changing form but not "quantity". Consequently, it is not possible to assert the existence of marginalist's notion of long-run equilibrium.

This criticism does not depend on aggregation problems and any other problems of marginalist theory. Some authors – like those mentioned in the introduction – confuse supply side critique of capital with those problems. The confusion concerns the aggregation of heterogeneous things like capital, labor, land and net output into a single unity, respectively. In that way, the treatment of each "factor of production" and of net output would be symmetrically the same.[16] Labor, land and capital can be expressed in physical terms. Despite that only the first two do not need to be aggregated, because its "[…] several endowments are sufficiently persistent to allow each of them to be treated as given […]" (Petri, 2004, p. 94). Garegnani (1978, p. 344, author's italics) is clear about this:

> In the case of factors of production which can be measured in physical terms, the choice of the unit of measurement is unimportant because a change of unit only implies multiplying the quantities by a *constant*. However, this is no longer true in the case of capital and other value magnitudes since the relative values of the commodities constituting alternative standards of value will themselves be different in different situations […].

Moreover, since the return on capital is measurable in value terms (not in physical terms) and the latter depends on prices and income distribution, the indeterminateness of the capital endowments emerges and, consequently, the aggregate production function becomes logically inconsistent. If there was only one good in the economy, the measurement in both physical and value terms would be the same, so that the price of the good and the price of capital would be symmetrical (Garegnani, 1970; Pasinetti, 2000, p. 403).

In Samuelson's (1962) model, the assumption that the composition of capital in all sectors is equal – that is, the conditions of production of both goods are the same – means that, in fact, there is

[16] That symmetry rises from the wrong "[…] realization that Ricardo's theory of intensive differential land implies a symmetry between land, and capital-cum-labor [sic] […], such that both can be seen as receiving their marginal product. It is then thought that the reasoning must be extensible to any source of income deriving from contributions to production: and therefore also to labor [sic] alone, and to capital alone" (Petri, 2004, p. 11).

only one good. Nevertheless, in that case, there would not be an aggregation problem, as noted by Garegnani (1970, p. 421). Besides, his model assumes that the capitalist uses only one type of capital good at a time for each chosen technique. Another technique is chosen when another type of capital good is used to produce consumption good – both goods are homogeneous.

There are two possibilities arising from the last assumption. First, this can mean that the several and different capital goods that can be used already exist in capital endowment. In other words, alternatives capital goods must already exist in the economy, and they cannot be used.[17] Thus, if there are alpha, beta, gamma, epsilon up to n alternative techniques in the economy, each with its own capital good, there would be $n-1$ capital goods not utilized in the initial capital endowment. However, Samuelson (1962) does not explain how the prices of these alternative capital goods, which can be adopted when distribution changes, are determined. This author does not explain whether the prices of these capital goods would be evaluated in terms of the current technique, or in terms of the technique that produces them but that no capitalist uses, or whether these prices would be zero, since at the moment when they are not being used there is no demand for them.[18] In the case where prices are positive, there is more room for the value of the capital endowment to change when there is a change in income distribution.

The second possibility is observed by Petri (2004, p. 219, 2021, p. 132): alternative capital goods to the capital good currently used to produce consumption good do not exist in the initial capital endowment. In this case, there must exist an alternative method of production that transforms the quantities of the currently used capital good (e.g., capital good Alpha), which constitutes the entire initial capital endowment, into another capital good (e.g., capital good Beta) that will be employed after the choice of techniques. Furthermore, there must also be a production method that transforms the capital good beta into capital goods gamma, epsilon, and so on, according to the sequence of periods. In general, the alternative capital good will only emerge when there is a method capable of transforming the currently used capital good into that alternative good.

However, these "transition methods" from one capital good to another must exist initially and would be used only once, until the alternative capital good becomes available for use. A problem arises: it is not possible to know the capital-labor ratio of these "transition methods", nor the costs of producing the alternative capital good that could be cost-minimizing. Both Samuelson (1962) and Garegnani (1970) did not analyze this "transition". In short, both possibilities are hard to imagine. Broadly speaking, the fact that income distribution is a determinant of value of capital instead of

[17] Furthermore, the existence of unused capital goods present in the capital endowment has implications for the demand-side critique, because there is more room for changes in the profit rate to produce a reversal in the direction of change in relative prices, and the change in the value of capital per worker can be large.

[18] The demand price is 0, therefore the capital good is not produced – its demand price is lower than the supply price. This is the same criticism made by Garegnani (1990) and Eatwell (1990) to Walras.

being determined by the latter, that is, initially available capital endowment is not a *datum* as marginalist theory requires, is a fundamental deficiency of latter theory and, however, has been neglected by marginalist theorists and some authors in the recent debates on wage curve's shape.

Final remarks

In the first instance, this essay aimed to explain as clearly as possible the criticisms developed throughout Controversies of Capital, specifically the discussion between Samuelson (1962) and Garegnani (1970). In this sense, a simple model of an economy that only has circulating capital and produces two basic goods was presented. From the model developed in Section 2, it was possible to understand that the conditions leading to a linear wage-profit curve and an inversely proportional relationship between the value of capital (per worker) and the rate of profit are quite restrictive. These conditions were the subject of discussion between Samuelson (1962) and Garegnani (1970). As shown, the surrogate production function cannot be a function, because there is not necessarily an inverse relationship between the prices of factors of production and the intensity with which they are used in the adopted methods. In other words, the wage-profit curve is not linear.

Furthermore, based on Baldone's (1984) criticisms, it became clear that the conditions for the existence of surrogate production function are more restrictive, even if both techniques share equal capital composition. Other criticisms of the Samuelson's (1962) model were raised, regarding the existence of fixed capital. His model does not clarify how the economy, which has fixed capital, reaches the state of full employment and remains in that position, when income distribution is changed and alters the chosen technique. Besides, to the extent that depreciation is not independent of profit rate, both supply-side and demand-side critiques of capital emerge.

Finally, it was possible to observe that Samuelson's (1962) surrogate production function is vulnerable not only to the more general critique regarding the indeterminateness of the capital endowment. His assumption that the capitalist uses only one type of capital good at a time for each technique adopted – the capital good is specific to each technique – implies two possibilities that are difficult to imagine. For all the reasons discussed in this essay, any attempt to rehabilitate the surrogate production function is meaningless, regardless of empirical estimates of the wage curve.

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