Probabilistic Failure Modeling of Pressure Relief Valves in Hydrogen Infrastructure via Bayesian Inference

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Abstract

Hydrogen is increasingly recognized as a strategic energy carrier in the transition toward low carbon energy systems, leading to the expansion of hydrogen production, storage, and distribution infrastructures. In high pressure hydrogen systems, pressure relief valves *(PRVs)* play a critical safety role by preventing overpressure conditions and protecting equipment integrity. However, hydrogen service introduces specific degradation mechanisms, particularly hydrogen embrittlement, which may reduce material toughness and increase the probability of valve malfunction. In addition, the limited availability of hydrogen specific failure data creates uncertainty in reliability estimation and complicates quantitative risk assessment *(QRA)* for hydrogen infrastructure. This study aims to develop probabilistic models for estimating the failure probabilities of *PRVs* operating in hydrogen environments. The proposed methodology applies a Bayesian reliability framework that integrates prior information from established industrial reliability databases with hydrogen operational evidence to update failure probability distributions. Three critical *PRV* failure modes are analyzed: failure to operate on demand, irregular operation, and external leak. Posterior probability distributions are obtained using appropriate combinations of Gamma, Lognormal, and Weibull models together with failure specific likelihood functions. The results indicate that failure on demand presents a probability of approximately 2.7%, external leak behaves as a rare event with low failure rates, and irregular operation shows the most significant reliability degradation over time. These findings demonstrate the value of Bayesian updating for addressing data scarcity and uncertainty in hydrogen systems. The proposed framework supports risk informed maintenance planning and provides quantitative reliability inputs for hydrogen specific *QRA* applications.

Keywords: hydrogen system, pressure relief valves, Bayesian inference, posterior distributions, quantitative risk assessment

1. Introduction

Hydrogen is rapidly emerging as a key component in the global transition to clean energy, with its adoption expanding across production, storage, and distribution infrastructures (Gao *et al.*, 2025). In high pressure hydrogen systems such as storage tanks and pipelines, pressure relief valves (PRVs) are crucial safety devices that regulate the flow of hydrogen when pressure exceeds safe levels (Tzioutzios *et al.*, 2025). However, Hydrogen causes embrittlement in PRV metals, reducing toughness and ductility, increasing crack formation risk under operational stress conditions (Campari *et al.*, 2023).

This embrittlement poses a significant reliability risk, which may lead to hidden defects that are difficult to detect but could deteriorate valve performance over time (Gao *et al.*, 2025). Empirical analysis show relief device failures significantly increase hydrogen system hazards, with PRVs contributing to many high pressure failure events, demonstrating that safety components can become critical failure points (Tzioutzios *et al.*, 2025). The complex failure mechanisms in hydrogen systems, including unexpected valve openings, valve seat wear, and cracking from embrittlement, emphasize the importance of thorough risk assessment and effective maintenance strategies (Campari *et al.*, 2023).

Traditional maintenance activity such as corrective maintenance, may not effectively address hydrogen specific risks, to prevent system failures, it's essential to implement maintenance strategies that ensure both reliability and safety, minimizing risks and extending component life (Cavalcante; Alencar; Lopes, 2017). Therefore, Proactive maintenance strategies based on inspection data, material degradation trends, and probabilistic failure modeling offer a more robust approach to improving system safety and operational reliability. In managing risks associated with pressure relief valve *(PRV)* failures, a systematic risk assessment is traditionally guided by three fundamental questions, what can go wrong? What are the potential consequences? And how frequently might such events occur? (Hosseini; Barker; Ramirez-Marquez, 2016). Building on this established framework, this present study extends the analysis by explicitly introducing an additional guiding question, how can the identified risks be reduced or contained?

Furthermore, recent advancement in dynamic risk analysis for hydrogen infrastructure demonstrate how probabilistic models such Bayesian or network based can be integrated to assess failure likelihood and predict maintenance needs for *PRV* under realistic operational uncertainty (Xiao *et al.*, 2024). The previous study addresses the first and third questions by identifying and classifying a *PRV* failure mechanisms and developing a probabilistic failure model corresponding to each identified mechanism. Building upon these developments the prior system level or maintenance focused analyses, this study explicitly formulates and updates probability distributions for each failure mechanism and formalizes the methodology through a step by step Bayesian flowchart, thereby providing quantitative reliability estimates with explicitly characterized uncertainty for direct integration into hydrogen specific *QRA*.

The objective of this study is to develop probabilistic models for estimating the failure probabilities of pressure relief valves *(PRVs)* in hydrogen systems through the development of failure mode specific probability distributions, thereby enhancing the accuracy and fidelity of quantitative risk assessment *(QRA)* for hydrogen production and distribution infrastructures.

The paper is structured as follows. Section 2 provides the description of the system and brief literature review. Section 3 presents the methodology and the modeling. Section 4 includes results and discussion. Section 5 concludes the paper.

2. Context of the system and brief literature review

2.1 System description

Pressure relief valves protect hydrogen systems from overpressure, but reliability is challenged by high demand, hydrogen embrittlement, and environmental conditions, making effective pressure control critical for safe and stable system operation. Hydrogen's unique molecular characteristics pose significant challenges for *PRV* design if not properly regulated, rapid discharge can lead to high excessive pressure the formation of a combustion wave development**,** significantly raising the risk of ignition (Jimenez; Groth, 2024).

Recent studies have shown that a new *PRV* specifically designed for hydrogen circulation systems can significantly improve pressure regulation performance by optimizing the flow pressure behavior under high pressure conditions (Wu *et al.*, 2025). Still, long term operational safety of *PRV* must also consider for the impact of material degradation. For instance, steels exposed to hydrogen may experience microstructural damage and crack initiation due to embrittlement mechanisms, highlighting the need for careful selection, testing, and maintenance of the pressure relief valve (Santana *et al.*, 2024).

In hydrogen systems, excessive pressure events can cause immediate valve failures or worsen hidden defects that remain undetected, progressing silently until reaching a critical state and potentially triggering catastrophic failure (Li; Torero; Guibaud, 2025). To prevent the failure of this system, it is essential to establish effective maintenance actions that ensure both reliability and safety. These actions help minimize the risk of unexpected breakdowns, extend the operational life of critical components, and maintain the system's ability to perform under demanding conditions (Rodrigues *et al.,* 2023).

Applying quantitative risk assessment (*QRA*) methodology has enabled the development of probabilistic models that integrate failure probabilities, failure rate analysis, and risk propagation behavior in hydrogen systems. These approaches highlight the importance of understanding both sudden failure modes and progressively evolving damage mechanisms in *PRV* (Ma *et al.*, 2024). Building on the probabilistic *PRV* failure modeling framework of Jimenez and Groth (2025), this study adds a clear methodological flowchart that documents the Bayesian updating process to improve transparency and reproducibility. It also extends prior work by evaluating reliability using posterior distributions and their percentiles, allowing a more robust assessment of *PRV* performance in hydrogen service. Since there is limited data for pressure relief valves in hydrogen system, prior information and likelihood functions are used to derive posterior distribution for the various failure modes.

Each failure mode can cause the system to transit from a functional state to a failed state. In determining the posterior estimates, both the failure rate (λ) and the probability of failure (ρ) can be used to quantify the likelihood of the system maintaining its functional condition and avoiding failure.

2.2 Brief literature review

Pressure relief valves (PRVs) serve as critical safety component for mitigating high pressure in hydrogen systems. Their reliability is essential not only for ensuring overall system safety but also for the accuracy of quantitative and probabilistic risk assessments (Lachance., 2009). Bayesian inference has emerged as a robust methodological framework for integrating limited operational data, test or bench scale observations, and expert judgment to develop posterior probability distributions for rare events and component failure rates, especially when direct failure data is scarce (Kelly; Smith, 2009). Within hydrogen infrastructure, Bayesian methods is use to update component failure frequencies, particularly for leak events, by combining generic failure rate data with limited hydrogen specific incident records (Kodoth *et al.*, 2020). NASA similarly uses Bayesian inference to refine event probability distributions in its Probabilistic Risk Assessment (*PRA*) programs, improving both the accuracy and robustness of system level risk evaluations (Stamatelatos *et al.*, 2011).

Beyond hydrogen systems, Bayesian methods are widely used in process industries to combine multiple data sources and produce robust failure rate estimates, especially for leak frequency analysis in LNG systems, and the approach is transferable to hydrogen equipment (Pörn *et al.*, 1996). For example, Bayesian inference has been used to refine probabilities of failure on demand and initiating event frequencies in Layers of Protection Analysis (*LOPA*) for natural gas (*LNG*) import terminals by integrating facility specific data with generic reliability databases. This process results in posterior distributions that enhance the accuracy of risk assessments (Yun; Rogers; Mannan, 2009). Likewise, Bayesian methods have been applied to estimate chemical release frequencies across multiple facilities using incident data from the U.S. Coast Guard's National Response Center, demonstrating the capability of Bayesian inference to incorporate historical multiyear data and generate improved probabilistic forecasts for various equipment types and failure mechanisms (Meel *et al.*, 2007).

Previous studies support Bayesian updating for hydrogen system reliability as data evolve, but PRVs remain underexamined. Existing work emphasizes leaks while overlooking other critical PRV failure modes. This gap motivates hydrogen specific probabilistic models covering multiple failures, enabling more accurate QRA and strengthening safety and reliability assessments for hydrogen infrastructure.

In response to this gap, this study develops hydrogen specific probability distributions for *PRV* failures using a Bayesian, failure mode specific framework. It moves beyond traditional leak focused analyses by modeling degradation mechanisms under hydrogen service, integrating available hydrogen data to generate refined distributions that enhance *QRA* accuracy and support evidence based safety and reliability decisions across hydrogen infrastructure systems.

3. Methodology

Numerous incidents and prior studies have documented operational and performance issues associated with pressure relief valves, prompting standardized classifications of *PRV* failure mechanisms. In particular, Center for Chemical Process Safety (2017) defines five failure modes, providing a structured framework for systematic safety and reliability analysis.

Table 1 – Failure modes of pressure relief valves (PRV)

	Failure Mode	Description
1	Failure to operate on demand	Failure to operate on demand refers to the inability of the valve to open correctly during an overpressure event, including delayed, incomplete, or insufficient opening to adequately relieve excess pressure.
2	Failure to re closed	Failure to reclose is the inability of the valve to properly reseat after pressure normalization, leading to continued leakage or discharge.
3	Irregular operation	Premature or spurious valve actuation in the absence of an overpressure condition, encompassing behavior functionally equivalent to internal or seat leakage.

4	External leak	This failure mode refers to the release of hydrogen to the environment through compromised connections, the valve body, or other structural components of the *PRV.*
5	Erratic output	Inadequate *PRV* behaviors. This failure mode encompasses unstable valve operation, including cycling and chattering characterized by rapid and repeated opening and closing of the *PRV.*

Source: Jimenez and Groth (2025)

The study focuses on three PRV failure modes failure to operate on demand, irregular operation, and external leakage because they are most directly associated with the primary safety function, loss of containment risk, and operational consequences in hydrogen systems. These failure modes have the biggest impact on safety and cost, and they can be reliably modeled using existing probabilistic methods even when hydrogen specific data are limited. In contrast, failure to reclose and erratic output are typically secondary or system interaction driven behaviors, often overlapping with leakage or operational instability and less suited to component level life distribution modeling. Therefore, the selected failure modes provide a practical and risk significant basis for Bayesian reliability assessment and maintenance optimization. Generic industry data define prior distributions, then hydrogen specific operational and incident data update them through Bayesian inference, producing uncertainty aware reliability estimates that strengthen risk and safety assessments.

3.1 Bayesian approach

Bayesian inference updates parameter estimates as new data becomes available, using an initial prior for θ and an adaptive framework that continuously refines posterior distributions as additional hydrogen specific data becomes available over time. This prior is then updated using observed data $D = \{d_1, d_2, d_3 \dots\dots d_n\}$ through the application of Bayesian' methods. The inference process is expressed as:

$$f_1(\theta|D) = \frac{f_0(\theta).L(D|\theta)}{\int f_0(\theta).L(D|\theta)d\theta} \quad (1)$$

Where $f_0(\theta)$ denotes the prior probability distribution of the parameter θ, $L(D|\theta)$ represents the likelihood function of the observed data given θ, and $f_1(\theta|D)$ is the posterior distribution reflecting the updated data after integrating the data (Li; Meeker, 2014). Bayesian inference is applied to estimate probability distribution parameters describing uncertainty in PRV time to failure for each failure mode, providing a statistically consistent, data driven representation of reliability behavior in hydrogen systems.

3.2 Data source and prior estimation

According to Jimenez and Groth (2025), the prior parameters were developed using non hydrogen *PRV* failure data from established reliability databases such as OREDA, CCPS, SINTEF PDS, and API RP 581. Historical data for demand failure, irregular operation, and external leaks were aggregated to form empirical prior distributions that represent variability in PRV performance across industries.

Parametric distributions specifically Gamma and Lognormal distributions were fitted to empirical prior data using maximum likelihood estimation to best represent observed characteristics. The estimated parameters (α, β, μ, σ, κ, Λ) define optimized prior models, which serve as the baseline distributions for subsequent Bayesian updating with hydrogen specific operational data.

Table 2 – Prior to being used in the hydrogen update process

Failure Mode	Prior Distribution	Variable Parameters		
Failure to operate on demand (demand based)	Gamma	ρ	α	36.466
			β	7.44×10^{-4}
Irregular operation	Gamma	$\alpha^{-\beta}$	k	6.168
			Λ	1.23×10^{-10}
External leaks	Lognormal	λ	μ	-16.415
			σ	0.593

Source: Jimenez and Groth (2025)

3.3 Hydrogen update process

This study develops hydrogen specific probability distributions for pressure relief valve failure modes using Bayesian updating that combines prior reliability data with hydrogen operational evidence. Prior distributions are first defined, then refined with observed data to produce posterior models that better represent hydrogen service conditions, improving probabilistic characterization of *PRV* failure behavior and uncertainty.

3.3.1 Failure to operate on demand

To integrate expert judgement and historical industry data, the failure to operate on demand probability is assigned a Gamma prior distribution. These prior captures the uncertainty in the baseline reliability of the valve before integrating hydrogen specific observations.

$$p \sim Gamma(\alpha_0, \beta_0) \tag{2}$$

With density:

$$f_0(p) = \frac{1}{\Gamma(\alpha_0){\beta_0}^{\alpha_0}} p^{\alpha_0 - 1} e^{-p/\beta_0} \tag{3}$$

The observed hydrogen system demands are modeled as Bernoulli trials where each demand results in either a success or a failure of the valve. Under this assumption, the probability of observing k failures in n demands follow a Binomial likelihood.

$$L(D|P) = \binom{n}{k} p^k (1-p)^{n-k} \tag{4}$$

The posterior updates the prior data using the observed hydrogen reliability data. Multiplying prior and likelihood yields the unnormalized posterior, which represents the updated but unscaled data about the true failure on demand probability.

$$f_{unnorm}(p) = f_o(p)\, L(D|P) \tag{5}$$

Since the failure on demand probability p is defined on the interval [0,1], numerical normalization is required to obtain a valid posterior distribution. Because the Gamma and Binomial model is non conjugate, the normalization constant is calculated over the interval p ∈ [0,1] using trapezoidal integration. This ensures that the posterior distribution integrates to one and remains consistent with the probabilistic meaning of p.

$$Z = \int_0^1 f_{unnorm}(p)\, dp \tag{6}$$

$$f_1(p|D) = \frac{f_{unnorm}(p)}{Z} \tag{7}$$

The posterior mean provides the expected failure on demand probability after considering hydrogen data. Due to the numerical posterior, the mean is also computed numerically.

$$\mathbb{E}[p|D] = \int_0^1 p\, f_1(p|D)\, dp \tag{8}$$

Because the posterior distribution is represented discretely, sampling on the normalized probability grid is used to quantify uncertainty through variance and selected posterior quantiles (5%, 50%, and 95%). These uncertainty measures characterize the variability in the failure on demand probability and provide a basis for subsequent reliability and risk assessments.

3.3.2 External leak

In the absence of hydrogen specific leak data, uncertainty in the failure rate (λ) is modeled using a lognormal prior distribution based on industry practice and expert knowledge from hydrogen PRV reliability literature.

$$\lambda \sim Lognormal(\mu_0 \sigma_0^2) \tag{9}$$

With density:

$$f_0(\lambda) = \frac{1}{\lambda\sigma_{prior}\sqrt{2\pi}} exp\left(-\frac{(\ln\lambda-\mu_{prior})^2}{2\sigma^2{}_{prior}}\right) \tag{10}$$

External hydrogen leaks are modeled as a Poisson process with constant hazard rate λ, consistent with standard assumptions for rare pressure boundary failures. Under this model, the contribution of observed failures and total exposure yields the exponential Poisson likelihood where r is the number of observed external leak and T is the total exposure time.

$$L(D|\lambda) = \lambda^r \exp(-\lambda\, T) \tag{11}$$

Since non conjugate lognormal and exponential model requires numerical normalization, which is performed using trapezoidal integration to ensure a valid posterior distribution.

$$f(\lambda|D) = \frac{1}{z} f_0(\lambda)\lambda^r \exp\left(-\lambda T_{total}\right) \tag{12}$$

The posterior mean represents the expected value of the failure rate after incorporating the available evidence and is widely used in reliability engineering to support maintenance planning and quantitative risk assessment. It is obtained by weighting all plausible values of the failure rate by their posterior probabilities, and is defined as

$$\mathbb{E}[\lambda] = \int_0^\infty \lambda f(\lambda|D) d\lambda \tag{13}$$

Posterior variance measures remaining uncertainty in the updated failure rate. Quantiles (e.g., 5%, 50%, and 95%), summarize this uncertainty, and mission time reliability is evaluated using the exponential survival function based on the posterior estimate.

$$R(T) = e^{-\lambda T} \tag{14}$$

which links the uncertainty in λ directly to uncertainty in system survival probability.

3.3.3 Irregular operation

To characterize the initial uncertainty associated with the irregular operation failure rate, a Gamma prior distribution is used. This prior reflects historical reliability data and ensures conjugacy with the Weibull likelihood, enabling closed form Bayesian updating.

$$\theta \sim Gamma\ (k_0 \Lambda_0) \tag{15}$$

With density:

$$f_0(\theta) = \frac{\Lambda_0^{k_0}}{\Gamma(k_0)} \theta^{k_0 - 1} e^{-\Lambda_0 \theta} \quad (16)$$

The Weibull model effectively captures degradation in irregular operations, with the likelihood function representing the probability of observing failures based on cumulative exposure. This is represented by T_β.

$$L(D|\theta) = \theta^r \exp(-\theta\, T_\beta) \quad (17)$$

Bayesian inference integrates prior knowledge with hydrogen specific data, and the conjugacy of the Gamma prior with the Weibull likelihood ensures the posterior remains analytically tractable.

$$\theta|D \sim Gamma(\, k_{post,}\ \Lambda_{post}) \quad (18)$$

The Weibull scale parameter governs characteristic life. Since $\theta = \alpha^{-\beta}$, the posterior uncertainty in θ can be translated into uncertainty in α, which is directly interpretable as $\alpha = \theta^{-1/\beta}$. The result in presented by reporting the mean, 5%, 50% and 95% quantiles of the new hydrogen α. The predictive reliability represents the expected probability that no irregular operation occurs before a specified time *T*, while explicitly accounting for uncertainty in the model parameters. This measure is obtained by integrating the reliability function over the full posterior distribution of the parameter *θ*, thereby jointly capturing stochastic failure behavior and parameter uncertainty. The resulting posterior predictive reliability,

$$\mathbb{E}[R(T)|D] = \left(\frac{\Lambda_{post}}{\Lambda_{post} + T^\beta}\right)^{k_{post}} \quad (19)$$

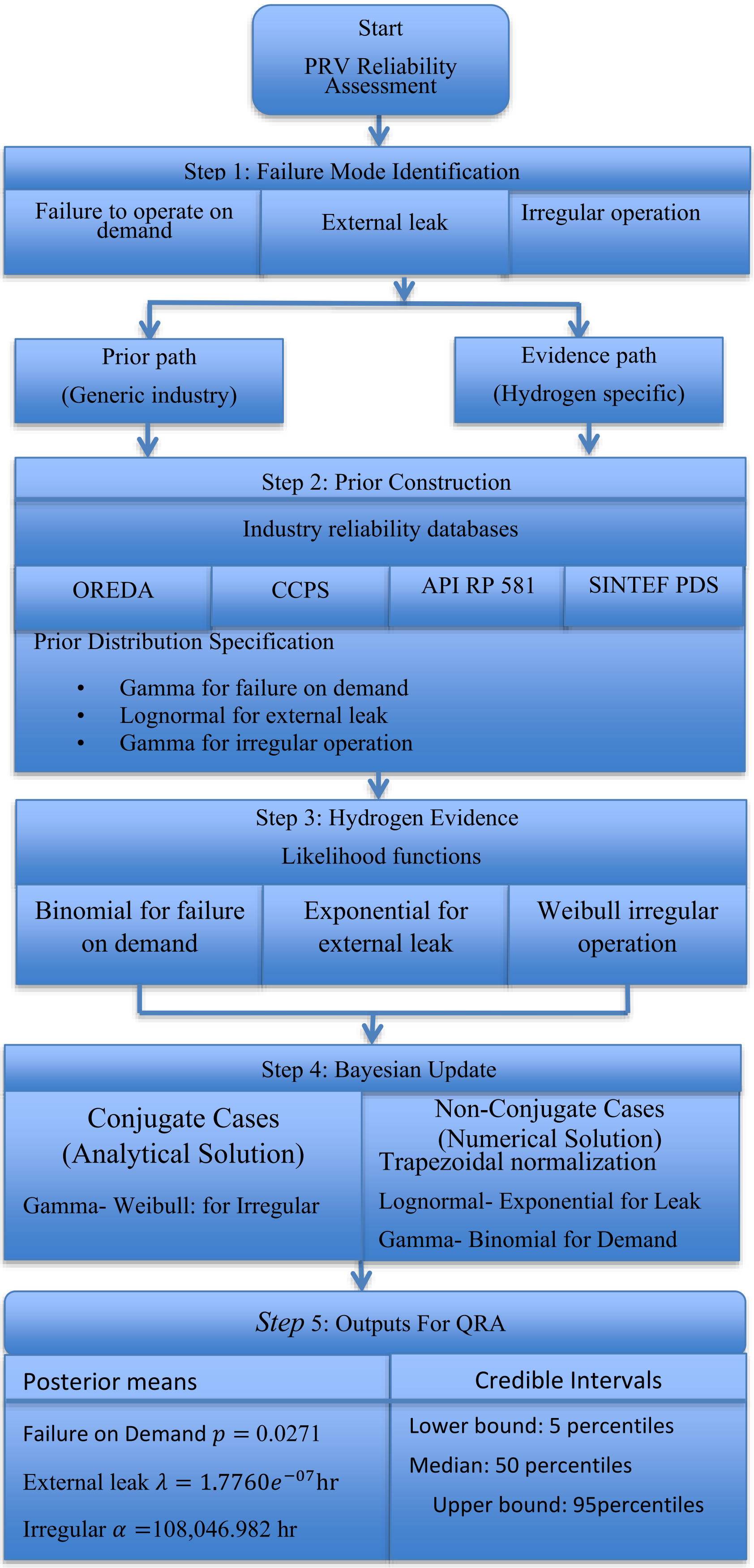
provides a statistically rigorous estimate of system survival that extends beyond point based reliability measures. As such, it offers an improved representation of *PRV* performance under irregular operation and supports more robust, risk informed reliability assessments for hydrogen systems.

The selected probability distributions reflect both *PRV* failure physics and the structure of available data. The Gamma distribution models uncertainty in positive failure rate parameters and allows flexible variance, making it more realistic than Normal or Uniform options. The Lognormal distribution captures external leak variability driven by multiple interacting factors, producing right skewed behavior over wide ranges. The Weibull distribution represents irregular operation because it can model aging and degradation trends, unlike the constant rate Exponential model. The Binomial distribution suits failure on demand, where each demand results in success or failure. The Exponential distribution provides a practical baseline for rare, random leak events. Each distribution was selected because it matches the physical failure mechanisms of *PRVs* in hydrogen system, operational characteristics of *PRVs*, the structure of the available data, and the requirements of statistically consistent Bayesian updating.

3.4 Methodological flowchart

The figure below presents a structured Bayesian framework for estimating *PRV* failure rates in hydrogen systems, starting from failure mode identification and prior definition, followed by incorporation of hydrogen specific data through appropriate likelihood functions. Bayesian updating using analytical or numerical methods produces posterior failure rate distributions with quantified uncertainty, which can be directly applied in quantitative risk assessment.

Figure 1 – Methodological flowchart



Source: Author (2025)

3.5 Step by step Bayesian updating algorithm pseudocode for PRV failure modeling

Figure 2 – Bayesian updating algorithm

Algorithm 1: Bayesian updating algorithm for PRV failure modelling

INPUT : **Prior:** Failure on Demand: Gamma(α_0, β_0); Leak: Lognormal(μ_0, σ_0); Irregular: Gamma(κ_0, Λ_0).
Data: Failure on Demand: $x = 3,\ n = 112$; External Leak: $(T_1 = 1.75, N_1 = 1),\ (T_2 = 105120, N_2 = 1)$; Irregular: $r = 5,\ \hat{\alpha} = 1.1723,\ \beta_w = 1.6$.
Grids: $n_p = 20000,\ n_\lambda = 5000,\ n_\theta = 200000$.
OUTPUT: Posterior PDFs, posterior means, credible intervals $(5\%, 50\%, 95\%)$, posterior predictive reliability.

`// Helper: trapezoidal numerical integration`
Function Trapz$(x[1:m], y[1:m])$:
  $A \leftarrow 0$
  for $i \leftarrow 1$ **to** $m-1$ **do**
    $A \leftarrow A + \frac{(x_{i+1} - x_i)}{2}(y_i + y_{i+1})$
  return A

`// Helper: weighted quantile for credible intervals`
Function WeightedQuantile$(x[1:m], w[1:m], q)$:
  $w \leftarrow w / \sum_{i=1}^{m} w_i$ `// normalise weights`
  $F \leftarrow 0$
  for $i \leftarrow 1$ **to** m **do**
    $F \leftarrow F + w_i$
    if $F \geq q$ **then**
      return x_i
  return x_m

`// (1) Failure on Demand: Gamma prior + Binomial likelihood (Non conjugate)`
Create uniform grid $p[1:n_p]$ over $[0,1]$ `// discretise failure prob.`
$\pi(p_i) \leftarrow$ GammaPDF$(p_i; \alpha_0, \beta_0)$ `// Gamma prior`
$L(p_i) \leftarrow \binom{n}{x} p_i^x (1-p_i)^{n-x}$ `// binomial likelihood`
$\tilde{\pi}(p_i|x,n) \leftarrow \pi(p_i) \cdot L(p_i)$ `// unnormalised posterior`
$Z_p \leftarrow$ Trapz$(p, \tilde{\pi}(\cdot|x,n))$ `// normalising constant`
$\pi(p_i|x,n) \leftarrow \tilde{\pi}(p_i|x,n)/Z_p$ `// normalised posterior PDF`
$\mathbb{E}[p|x,n] \leftarrow$ Trapz$(p, p \cdot \pi(p|x,n))$ `// posterior mean`
$p_{0.05} \leftarrow$ WeightedQuantile$(p, \pi(p|x,n), 0.05)$ `// 5% bound`
$p_{0.50} \leftarrow$ WeightedQuantile$(p, \pi(p|x,n), 0.50)$ `// median`
$p_{0.95} \leftarrow$ WeightedQuantile$(p, \pi(p|x,n), 0.95)$ `// 95% bound`
$R_{\mathrm{FTOD}} \leftarrow 1 - \mathbb{E}[p|x,n]$ `// posterior predictive reliability`

`// (2) External Leak: Lognormal prior + Exponential likelihood (Non conjugate)`
$E \leftarrow \sum_j N_j T_j$ `// total exposure time`
$k \leftarrow \sum_j N_j$ `// total leak events (k = 2)`
Create grid $\lambda[1:n_\lambda]$ `// discretise rate parameter`
$\pi(\lambda_i) \leftarrow$ LogNormalPDF$(\lambda_i; \mu_0, \sigma_0)$ `// lognormal prior`
$L(\lambda_i) \leftarrow (\lambda_i E)^k e^{-\lambda_i E}$ `// Exponential likelihood`
$\tilde{\pi}(\lambda_i|k,E) \leftarrow \pi(\lambda_i) \cdot L(\lambda_i)$ `// unnormalised posterior`
$Z_\lambda \leftarrow$ Trapz$(\lambda, \tilde{\pi}(\cdot|k,E))$ `// normalising constant`
$\pi(\lambda_i|k,E) \leftarrow \tilde{\pi}(\lambda_i|k,E)/Z_\lambda$ `// normalised posterior`
$\mathbb{E}[\lambda|k,E] \leftarrow$ Trapz$(\lambda,\ \lambda \cdot \pi(\lambda|k,E))$ `// posterior mean rate`
$R_{\mathrm{Leak}}(t) \leftarrow \exp(-\mathbb{E}[\lambda|k,E]\,t)$ `// predictive reliability at t`

`// (3) Irregular Operation: Gamma prior + Weibull likelihood (conjugate)`
$S \leftarrow \sum_{i=1}^{r} t_i^{\beta_w}$ `// sufficient statistic`
$\kappa_1 \leftarrow \kappa_0 + r$ `// posterior shape`
$\Lambda_1 \leftarrow \Lambda_0 + S$ `// posterior rate`
$\mathbb{E}[\theta] \leftarrow \kappa_1/\Lambda_1$ `// posterior mean`
$R_{\mathrm{Irregular}}(t) \leftarrow \int_0^\infty \exp(-\theta t^{\beta_w})\,\pi(\theta)\,d\theta$ `// predictive reliability (quadrature)`
return posterior distributions, summary statistics, reliability functions

Source: Author (2025)

The framework applies adaptive Bayesian updating to PRV failure rates, integrating industry and hydrogen specific data. It supports analytical and numerical inference, preserves full posterior uncertainty, and provides a consistent, traceable basis for quantitative risk assessment under limited, evolving data conditions.

4. Results and Discussion

This section presents Bayesian posterior life distributions for PRV failure modes in hydrogen systems, derived using the methodology in Section 3 and data from Table 2.

Table 3 shows posterior failure probabilities, while Table 4 summarizes Bayesian parameter distributions with 5%, median, and 95% life percentiles.

Table 3 – Posterior mean and parameter estimate for PRV failure modes in hydrogen systems

Failure Mode	Life Distribution	Mean		Posterior Parameter Estimate	
Failure to operate on demand	Binomial	p	0.02706	α	39.5868
				β	6.837515×10^{-04}
External leak	Exponential	λ	$1.7760e^{-07}$hr	μ	-15.718
				σ	0.591
Irregular operation	Weibull	α	108,046.982hr	k	11.168
				Λ	1.172×10^{-09}

Source: Author (2025)

Table 4 – Posterior distributions and percentile estimate of failure parameters for PRV failure modes

Failure Mode	Posterior Distribution	Posterior Percentile Estimates			
		P	Lower	Median	Upper
Failure to operate on demand	Gamma	p	$2.0421E^{-02}$	$2.6821E^{-02}$	$3.4562E^{-02}$
External leak	Lognormal	λ	$5.6435E^{-08}$	$1.4949E^{-07}$	$3.9420E^{-07}$
Irregular operation	Gamma	α	78,775.603	105,037.404	147,357.400

Source: Author (2025)

Table 5 provides practical insight into how each PRV failure mode affects real world system performance and maintenance strategy. The failure to operate on demand (p = 0.02706) indicates that the valve has approximately a 2.7% probability of failing when required, meaning reliability is high per demand but dependent on functional testing rather than operating time. In contrast, external leak exhibits very slow degradation, maintaining high reliability even at long mission times (above 96% at 100,000 hours in the conservative case), suggesting it is a long term integrity concern rather than an immediate operational threat. However, irregular operation shows rapid reliability decline, dropping to nearly 41% at 100,000 hours under the mean estimate and as low as 23% in the conservative percentile case, indicating substantial uncertainty and making it the dominant driver of maintenance planning decisions. Overall, the percentile estimates highlight the range between optimistic and conservative states, supporting risk informed inspection intervals and demonstrating that irregular operation represents the most critical mechanism for long term reliability and economic optimization in hydrogen PRV systems.

Table 5 – Posterior and percentile reliability estimates for PRV failure modes

Failure mode	Posterior mean	Mission time	Mean reliability	5% percentile	Median reliability	95% percentile
Failure to operate on demand	$p = 0.02706$	On demand	0.9729	0.9795	0.9732	0.9654
External leakage	$\lambda = 1.78e^{-07}hr$	50,000hr	0.9911	0.9971	0.9925	0.9804
		100,000hr	0.9823	0.9943	0.9851	0.9613
Irregular operation	$\alpha = 1.08\ e^{05}hr$	50,000hr	0.7471	0.6165	0.7369	0.8374
		100,000hr	0.4131	0.2307	0.3964	0.5840

Source: Author (2025)

4.1 Numerical study and analysis

This numerical case study assesses the reliability of a pressure relief valve operating in high pressure hydrogen service using failure mode specific Bayesian models, while accounting for uncertainty due to limited hydrogen data. Failure on demand is updated using a Gamma prior and Binomial likelihood, giving a posterior mean probability p of 0.02706. External leakage is modeled as a rare event with a lognormal failure rate, with a mean of 1.776×10^{-7} per hour. Irregular operation is represented using a Weibull life model with a characteristic life parameter α = 108,046.982 hours, capturing degradation related behavior. Together, these results provide hydrogen specific reliability inputs suitable for quantitative risk assessment.

5. Final considerations

This study developed an adaptive Bayesian reliability framework for pressure relief valves operating in hydrogen systems, where failure data are limited. Failure mode specific probability distributions were established for failure to operate on demand, irregular operation, and external leakage by combining generic industrial reliability data with available hydrogen evidence. The inclusion of a clear methodological flowchart and a step by step updating procedure improves transparency, consistency, and reproducibility, and supports direct use in quantitative risk assessment. The results show that irregular operation leads to faster reliability degradation, external leakage behaves as a rare event process, and demand failure is time independent.

Despite these contributions, the study is constrained by limited hydrogen specific data, simplifying assumptions such as constant failure behavior and independent failure modes, and the exclusion of reclosure and erratic output failures. To address the question of how the identified risks can be reduced or contained, the study indicates that risk mitigation should focus on structured, reliability based maintenance strategies. These include the implementation of risk informed inspection intervals derived from probabilistic reliability models, continuous condition monitoring to detect early signs of degradation, and improved material selection to limit hydrogen embrittlement effects. Furthermore, adaptive maintenance planning, in which inspection and maintenance decisions are updated as new operational data become available, enables progressive refinement of reliability estimates and risk controls. In future work, explicitly incorporating optimized periodic inspection intervals into the framework will further strengthen maintenance optimization and enhance the long-term safety and reliability of hydrogen systems.

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