Quantum Coherences in the Thermal Relaxation Asymmetry

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***Abstract*— In this work, we studied the asymmetry in thermal relaxation of a qubit interacting weakly with a thermal bath of harmonic oscillators. Our propose is to answer the question if the presence of coherence in the initial state of the qubit affects heating or cooling it. Using the theory of open quantum systems and thermal kinematics, our results showed that heating may be faster than cooling, and that coherences may delay the relaxation process.**

***Keywords*— Relaxation, Asymmetry, Coherence**

I. INTRODUCTION

According to thermodynamics, a system interacting with a thermal bath will evolute to the bath's temperature through thermal relaxation. For states near the equilibrium, Onsager proposed the regression hypothesis, in which the mean behaviour of statistical fluctuations is the same of the macroscopic system: linear, memoryless and symmetric [1]. This symmetry is related to reversibility, meaning that the system travels the same thermodynamic trajectory to go to a state and come back to the initial state.

However, this hypothesis fails for quantum systems [2] and for systems initially far-from-equilibrium [3], due to the loss of linearity and the possibility of becoming asymmetric. Recently, experiments showed the existence of an asymmetry for out-of-equilibrium classical systems, in which heating is faster than cooling [4], [5]. Still, it is important to notice that this asymmetry is not universal for systems with discrete energy levels, existing the possibility of cooling becoming faster than heating under certain conditions [6].

This asymmetric behaviour also exist for quantum systems [7], [8]. In the quantum regime, there is a diversity of phenomena due to quantum correlations. It is known that the presence of these correlations creates thermodynamic consequences, as the anomalous heat exchange [9], [10] and non-classical work extraction [11].

Therefore, the question if and how quantum correlations affects the asymmetry in heating and cooling naturally arises. Because the coherence is a necessary condition for quantum correlations in a system, this work aims to study the thermal relaxation of systems with coherence. In order to answer this question, we used the theory of open quantum systems to obtain the dynamics of one qubit in contact with a thermal bath made of harmonic oscillators. Then, we computed the velocity and the degree of completion described by the thermal kinematics [4], [6], [8], [12]. This two quantities are essential to verify and quantify the asymmetry in thermal relaxation.

This project if funded by CAPES (88887.000132/2024-00).

II. FIGURES

The Figure 1 show the the degree of completion for different initial coherences. Note that heating is always closer to equilibrium. Also, the increase of the coherence result in the initial state moving away from the equilibrium.

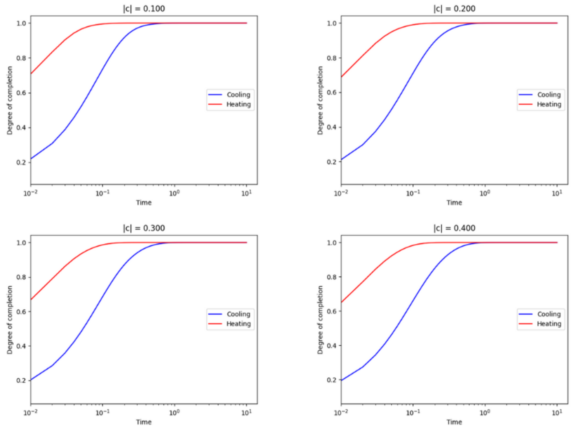

Fig. 1. Degree of completion of the thermodynamic trajectory for different initial coherences.

The Figure 2 show the velocity of the process as a function of time. According to [4], the initial higher velocity of heating along with the proximity of equilibrium is enough to make heating faster than cooling. Notice that the increasing coherence results in increasing initial velocity.

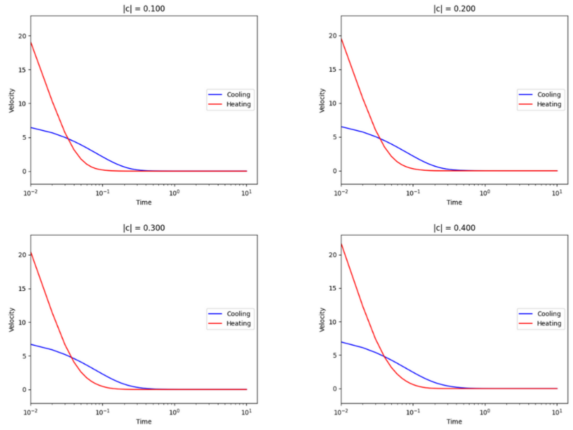

Fig. 2. Velocity for different initial coherences.

The Figure 3 show the degree of completion for heating and cooling of systems with different initial coherences. We observed that heating reaches equilibrium before cooling. Also, systems with smaller initial coherence reaches equilibrium faster.

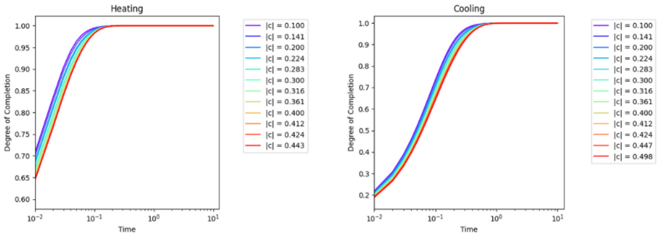

Fig. 3. Degree of completion for heating and cooling systems with different coherences.

III. EQUATIONS

We considered one qubit as the systems of interest, $H_S = \frac{\omega_0}{2}\sigma_z$, where ω_0 is the energy gap of the qubit levels and σ_z is the Pauli matrix for the z axis. The initial density matrix,

$$\rho(0) = \begin{bmatrix} p & C \\ C^* & 1-p \end{bmatrix}, \tag{1}$$

have initial coherence given by the parameter C and populations given by

$$p = \frac{e^{-\beta_0\omega_0/2}}{2\cosh(\beta_0\omega_0/2)}, \tag{2}$$

in which $\beta_0 = 1/k_B T_0$ with T_0 being the temperature of the qubit.

The hamiltonian of the thermal bath made of harmonic oscillators is given by $H_B = \sum_k \omega_k b_k^\dagger b_k$, where ω_k indicates the frequency of k^{th} harmonic oscillator, $b_k^\dagger$ and b_k are the creation and annihilation operators. Using a bath with temperature T ($\beta = 1/k_B T$), the density matrix of the bath is

$$\rho_R = \prod_k (1 - e^{-\beta\omega_k}) e^{-\beta\omega_k b_k^\dagger b_k}. \tag{3}$$

In this work, we considered a weak coupling between the qubit and the thermal bath, that interact according to the hamiltonian in the interaction picture

$$H_I(t) = \sum_k g_k(\sigma_- b_k^\dagger e^{-i(\omega_0-\omega_k)t} + \sigma_+ b_k e^{i(\omega_0-\omega_k)t}), \tag{4}$$

being g_k the coupling parameter.

Combining these informations, we solved the Redfield Equation to obtain the dynamics of the system. The elements of the density matrix are

$$\rho_{00}(t) = \frac{\bar{n}_{th}+1}{2\bar{n}_{th}+1} + \left(p - \frac{\bar{n}_{th}+1}{2\bar{n}_{th}+1}\right) e^{-2\Gamma(2\bar{n}_{th}+1)t}, \tag{5}$$

$$\rho_{11}(t) = \frac{\bar{n}_{th}}{2\bar{n}_{th}+1} + \left(\frac{\bar{n}_{th}+1}{2\bar{n}_{th}+1} - p\right) e^{-2\Gamma(2\bar{n}_{th}+1)t}, \tag{6}$$

$$\rho_{01}(t) = C e^{-2\Gamma(\bar{n}_{th}+1/2)t} \tag{7}$$

$$\rho_{10}(t) = C^* e^{-2\Gamma(\bar{n}_{th}+1/2)t}, \tag{8}$$

Rewriting this equations in the Bloch representation, we can compute the quantum Fisher information using the expression $\mathcal{I}_Q = |\partial_t r|^2 + (r\cdot\partial_t r)^2/(1-|r|^2)$. Following [4], we compute the velocity (9), the position (10) and the degree of completion (11) (which indicates the distance from equilibrium).

$$v(t) = \frac{1}{2}\sqrt{\mathcal{I}_Q(\rho(t))} \tag{9}$$

$$\mathcal{L}(t_i, t_f) = \frac{1}{2}\int_{t_i}^{t_f} \sqrt{\mathcal{I}_Q(\rho(t))} \tag{10}$$

$$\varphi(s) \equiv \frac{\mathcal{L}(t_i, t_s)}{\mathcal{L}(t_i, t_f)} \tag{11}$$

IV. CONCLUSIONS

Our results showed that the increase of coherence generates an increase in the initial velocity and a decrease in the initial distance from equilibrium. The degree of completion showed that heating seems faster than cooling and the presence of coherence may delay the system to reach equilibrium. More analyses are being made to understand the behaviour of information flux between the bath and the system.

ACKNOWLEDGEMENTS

Thanks to CAPES for funding this project. Also thanks to all the members of the Group of Quantum Information from IFSC-USP for the discussions.

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