Experimental Demonstration of Theory-Independent Context Incompatibility using Single-Photon Qubits

Patrick Lima, Mariana Storrer, Ana C. S. Costa, Renato M. Angelo, Sebastião Pádua

Abstract— **In this work, we present the detailed experimental implementation of a test for "theory-independent context compatibility," a principle satisfied by classical physics but violated by quantum mechanics. We focus on a single-photon quantum optics platform to verify this violation. We describe the preparation of arbitrary single-photon qubit mixed states via spontaneous parametric down-conversion (SPDC) and the realization of sequential non-selective measurements using polarizing beam splitters and wave plates. Our results, obtained from a robust optical apparatus, confirm quantum predictions and demonstrate a pronounced degree of incompatibility, offering a practical validation of a generalized notion of incompatibility.**

Keywords— **Quantum Optics, Experimental Quantum Information, Context Incompatibility, General Probabilistic Theories**

I. Introduction

Measurement incompatibility is a cornerstone of quantum foundations, distinguishing the quantum and classical realms. Historically linked to the uncertainty principle and the non-commutativity of observables, the notion has evolved to the concept of the absence of joint measurability, applicable to generalized measurements (POVMs). Far from being just a limitation, measurement incompatibility is the resource that underpins crucial quantum tasks, such as quantum state discrimination and cryptography, and is intrinsically linked to resources like non-locality and contextuality. Given the possibility that quantum mechanics may not be nature's most fundamental theory, investigations into incompatibility in more general theories are crucial. Recent works have introduced the notion of *context incompatibility*, which incorporates the quantum state into its definition and describes the transition to the classical limit induced by decoherence [24]. Aiming to generalize even further, a *theory-independent context incompatibility* (TICI) was proposed, which applies to generic probabilistic theories [37].

This paper focuses on the experimental implementation and verification of this TICI principle. While the theoretical framework is fundamental, our primary goal is to detail the practical realization of a test of this hypothesis. We describe a single-photon quantum optics experiment designed to: (i) precisely prepare single-qubit mixed states, (ii) implement sequential non-selective measurements of non-commuting observables, and (iii) use the measured probabilities to quantify the degree of violation of context compatibility. Our results confirm that nature violates context compatibility, aligning with the predictions of quantum mechanics and providing a tangible demonstration of a generalized notion of incompatibility.

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II. Theoretical Framework

To establish the foundation for our experiment, we first define the concept of context compatibility in a theory-independent manner [37]. We consider a context $\mathbb{C} = \{\mathscr{E}, \mathscr{X}, \mathscr{Y}\}$, which consists of a state preparation $\mathscr{E}$ and two generalized measurements $\mathscr{X}$ and $\mathscr{Y}$. A non-selective measurement of $\mathscr{X}$ on state $\mathscr{E}$ yields a new state $\mathscr{M}_{\mathscr{X}}(\mathscr{E})$, with the probability distribution for a subsequent measurement of $\mathscr{Y}$ given by $p_{\mathscr{M}_{\mathscr{X}}(\mathscr{E})}(y_j) = \sum_i p_{\mathscr{E}}(y_j|x_i)p_{\mathscr{E}}(x_i)$.

Definition. *A context $\mathbb{C} = \{\mathscr{E}, \mathscr{X}, \mathscr{Y}\}$ is said to be compatible if the non-selective measurement of one variable does not disturb the probability distribution of the other, and vice versa. Mathematiqually, this means:*

$$p_{\mathscr{E}}(x_i) = p_{\mathscr{M}_{\mathscr{Y}}(\mathscr{E})}(x_i) \quad \textit{and} \quad p_{\mathscr{E}}(y_j) = p_{\mathscr{M}_{\mathscr{X}}(\mathscr{E})}(y_j), \tag{1}$$

This criterion is trivially satisfied in classical statistical mechanics, where the order of non-destructive measurements does not alter the underlying phase-space probability distribution. However, as quantum mechanics does not admit joint probability distributions for arbitrary observables, a violation of this principle is expected. In the quantum formalism, for a context $\mathbb{C} = \{\rho, A, B\}$ with projective measurements, the compatibility condition (1) is equivalent to the following operator conditions:

$$\Phi_A(\rho) = \Phi_{AB}(\rho) \quad \text{and} \quad \Phi_B(\rho) = \Phi_{BA}(\rho), \tag{2}$$

where $\Phi_A(\rho) = \sum_i (A_i \rho A_i)$ is the non-selective measurement map, with A_i being the projectors of observable A. To quantify the violation of compatibility, the *theory-independent context incompatibility* (TICI) measure is used, based on the Kullback-Leibler divergence [37]:

$$\mathcal{I}_{\mathbb{C}} := \frac{D(P_{\mathscr{E}}(\mathscr{X}) \| P_{\mathscr{M}_{\mathscr{Y}}(\mathscr{E})}(\mathscr{X})) + D(P_{\mathscr{E}}(\mathscr{Y}) \| P_{\mathscr{M}_{\mathscr{X}}(\mathscr{E})}(\mathscr{Y}))}{2}. \tag{3}$$

For the quantum case with projective measurements, this translates into an expression involving von Neumann's relative entropy:

$$\mathcal{I}_{\{\rho,A,B\}} = \frac{S(\Phi_A(\rho) \| \Phi_{AB}(\rho)) + S(\Phi_B(\rho) \| \Phi_{BA}(\rho))}{2}. \tag{4}$$

Our experiment aims to measure the necessary probability distributions to calculate $\mathcal{I}_{\mathbb{C}}$ and test its behavior as a function of the initial quantum state ρ.

III. Experimental Implementation

The core of our experimental proposal is the implementation and control of each component of the quantum context $\mathbb{C} = \{\rho, A, B\}$ using the polarization degree of freedom of single photons.

A. Preparation of Arbitrary Single-Qubit States

To test the state-dependence of incompatibility, we need a reliable method to prepare an arbitrary single-qubit mixed state, parametrized as:

$$\rho = \frac{p}{2}\mathbb{1} + (1-p)\,|\psi\rangle\langle\psi|\,, \tag{5}$$

$$\text{where} \quad |\psi\rangle = \cos\left(\frac{\theta}{2}\right)|H\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|V\rangle\,. \tag{6}$$

The parameter $p \in [0,2]$ controls the state's purity, varying from a pure state ($p = 0$ or $p = 2$) to a maximally mixed state ($p = 1$). This is achieved using polarization-entangled photon pairs from SPDC. The unitary transformations for the relevant optical elements are given by a half-wave plate (HWP) and a quarter-wave plate (QWP):

$$U_{\text{HWP}}(\theta) = \begin{bmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{bmatrix}, \quad U_{\text{QWP}}(\phi) = \begin{bmatrix}1 & 0 \\ 0 & e^{i\phi}\end{bmatrix}. \tag{7}$$

A vertically polarized pump laser state $|V\rangle_{pump}$ is transformed by an HWP at angle θ_p into $\sin(2\theta_p)\,|H\rangle_{pump} - \cos(2\theta_p)\,|V\rangle_{pump}$. This pump beam generates a two-photon entangled state in two adjacent BiBO crystals:

$$|\Psi\rangle = \cos(2\theta_p)\,|HH\rangle + \sin(2\theta_p)e^{i\delta}\,|VV\rangle\,. \tag{8}$$

Detecting one photon (the "trigger") without resolving its polarization corresponds to a partial trace over its Hilbert space. This prepares the second photon (the "signal") in the mixed state:

$$\rho_0 = \text{Tr}_{\text{trigger}}(|\Psi\rangle\langle\Psi|) = \cos^2(2\theta_p)\,|H\rangle\langle H| + \sin^2(2\theta_p)\,|V\rangle\langle V|\,. \tag{9}$$

By comparing this with the target state form for $\theta = \phi = 0$, we find the relation $p = 2\sin^2(2\theta_p) = 1 - \cos(4\theta_p)$. This provides precise control over the state's purity via the pump HWP angle θ_p. To achieve an arbitrary pure state component $|\psi\rangle$, the signal photon subsequently passes through a HWP_1 and a QWP_1, which perform the desired SU(2) rotation, with $\theta = 4\theta_1$ and $\phi = \phi_1$.

B. Realization of Non-Selective Measurements

The experimental implementation of a non-selective measurement map $\Phi_A(\rho)$ requires a process that first measures the observable A in a non-demolition manner and then incoherently sums all possible outcomes. We achieve this by utilizing the optical path of the photon as an auxiliary degree of freedom (ancilla). The polarizing beam splitter (PBS) is the key element, implementing a controlled-NOT (CNOT) operation in the basis of the Hilbert space $\mathcal{H} = \mathcal{H}_{\text{pol}} \otimes \mathcal{H}_{\text{path}}$, where polarization acts as the control qubit and path as the target.

The goal is to implement the map $\Phi_A(\rho) = \sum_i A_i \rho A_i$, where for projective measurements $A_i = |a_i\rangle\langle a_i|$. This is achieved via a sequence of unitary operations. Let R_A be the unitary operation (implemented with wave plates) that maps the computational basis to the eigenbasis of A, i.e., $R_A\,|i\rangle = |a_i\rangle$. The full non-demolition measurement process is:

$$|a_i\rangle\,|0\rangle \xrightarrow{R_A^\dagger} |i\rangle\,|0\rangle \xrightarrow{\text{PBS}} |i\rangle\,|i\rangle \xrightarrow{R_A} |a_i\rangle\,|i\rangle\,. \tag{10}$$

To formally verify this process, we apply it to an arbitrary initial state $\rho \otimes |0\rangle\langle 0|$:

$$\begin{aligned}\rho \otimes |0\rangle\langle 0| &\xrightarrow{R_A^\dagger \otimes I} R_A^\dagger \rho R_A \otimes |0\rangle\langle 0| \\ &\xrightarrow{\text{PBS}} \sum_{i,j=0,1} |i\rangle\langle i|\, R_A^\dagger \rho R_A\,|j\rangle\langle j| \otimes |i\rangle\langle j| \\ &\xrightarrow{R_A \otimes I} \sum_{i,j=0,1} R_A\,|i\rangle\langle i|\, R_A^\dagger \rho R_A\,|j\rangle\langle j|\, R_A^\dagger \otimes |i\rangle\langle j| \\ &= \sum_{i,j=0,1} |a_i\rangle\langle a_i|\,\rho\,|a_j\rangle\langle a_j| \otimes |i\rangle\langle j| \equiv \rho_{\text{res}}.\end{aligned} \tag{11}$$

Finally, tracing out the path degree of freedom (ancilla) gives the desired map:

$$\text{Tr}_{\text{path}}\{\rho_{\text{res}}\} = \sum_i |a_i\rangle\langle a_i|\,\rho\,|a_i\rangle\langle a_i| = \Phi_A(\rho). \tag{12}$$

For our experiment with observables $A \in \{\sigma_x, \sigma_z\}$, we find the required HWP angles are $\theta_x = \frac{\pi}{8}$ and $\theta_z = 0$, with no QWPs needed for the measurement stage, which simplifies the setup.

C. Sequential Measurement Setup and Protocol

To calculate $\mathcal{I}_{\mathbb{C}}$, we need sequences like $\Phi_{BA}(\rho) = \Phi_B(\Phi_A(\rho))$, implemented by cascading two non-demolition measurement units. For our chosen observables $A = \sigma_x$ and $B = \sigma_z$, the setup only requires HWPs. The final experimental arrangement is shown in Fig. 1.

To simplify the apparatus, only the transmitted path through both PBSs is directed to a single-photon detector (D1). To measure all four outcomes of a sequential measurement (e.g., outcomes '00', '01', '10', '11'), we rotate the measurement HWPs (HWP_A, HWP_B) by 45°. This rotation swaps the roles of transmitted and reflected polarizations, allowing us to probe each outcome channel with a single detector. Coincidence counts between D1 and the trigger detector D2 are recorded.

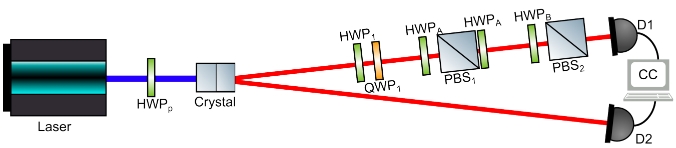

Fig. 1

Experimental scheme used to obtain the photon-counting probabilities. HWP_p, HWP_1, and QWP_1 prepare the initial state. HWP_As and PBS_1 perform the first non-selective measurement $A = \sigma_x$. HWP_B and PBS_2 perform the second non-selective measurement $B = \sigma_z$. Detector D1 is not polarization-sensitive, and counts are recorded in coincidence (CC) with detector D2.

D. From Photon Counts to Probabilities

The raw data from our experiment are the coincidence counts $C(a_i, b_j)$ corresponding to the outcome a_i for the first measurement and b_j for the second. The joint probabilities are obtained via normalization: $p(a_i, b_j) = C(a_i, b_j)/\sum_{i,j} C(a_i, b_j)$. The four probability distributions required for the TICI quantifier in Eq. (3) are extracted from these joint probabilities. For a sequence where observable A is measured first, followed by B, we have:

- The probability of outcome b_j after a non-selective measurement of A is obtained by summing over all outcomes of A: $p(b_j|\Phi_A(\rho)) = \sum_i p(b_j, a_i)$.
- The probability of outcome a_i on the initial state is obtained by summing over all outcomes of B: $p(a_i|\rho) = \sum_j p(b_j, a_i)$.

Analogous relations hold for the reverse measurement sequence (B then A), allowing us to determine $p(a_i|\Phi_B(\rho))$ and $p(b_j|\rho)$. This procedure provides all the necessary quantities to experimentally calculate $\mathcal{I}_{\mathbb{C}}$. The relative entropy was obtained using the following expression:

$$\begin{aligned} &S(\Phi_A(\rho)||\Phi_{AB}(\rho)) \\ &= \sum_i \langle a_i| \sum_j A_j\rho A_j |a_i\rangle \log_b\left(\frac{\langle a_i|\sum_j A_j\rho A_j|a_i\rangle}{\langle a_i|\sum_{kl} A_l B_k \rho B_k A_l |a_i\rangle}\right) \\ &= \sum_i \langle a_i|\rho|a_i\rangle \log_b\left(\frac{\langle a_i|\rho|a_i\rangle}{\langle a_i|\Phi_B(\rho)|a_i\rangle}\right) \\ &= \sum_i \mathrm{Tr}\,\rho A_i \log_b\left(\frac{\mathrm{Tr}\,\rho A_i}{\mathrm{Tr}\,\Phi_B(\rho)A_i}\right) \\ &= \sum_i p(a_i|\rho)\log_b\left(\frac{p(a_i|\rho)}{p(a_i|\Phi_B(\rho))}\right), \end{aligned} \tag{13}$$

where $A = \sum_{i=0}^{1} a_i A_i$ and $B = \sum_{i=0}^{1} b_i B_i$, with A_i and B_i being projectors on eigenstates with eigenvalues a_i and b_i respectively. This required the experimental implementation of projectors A_0, A_1, B_0, B_1. In our setup, each projector was parameterized by two half-wave plate angle, which are subject to small experimental errors. Furthermore, each pair of projectors ($A_i B_j$ or $B_i A_j$) was implemented in a distinct "run" of the experiment. This means that the experimentally implemented projectors A_0, A_1 and B_0, B_1 may not perfectly sum to identity, nor are they perfectly orthogonal due to the experimental uncertainty. These random errors affect the theoretical curve since it is calculated supposing a perfect observable implementation.

IV. Results and Discussion

Using the described apparatus and protocol, we measured the photon counts for both measurement sequences. From these, we calculated the required probability distributions and evaluated the context incompatibility $\mathcal{I}_{\mathbb{C}}$. We performed three experimental runs, each time aligning the pure-state component of ρ with the x, y, or z axis of the Bloch sphere, while varying the mixture parameter p. The results are shown in Fig. 2.

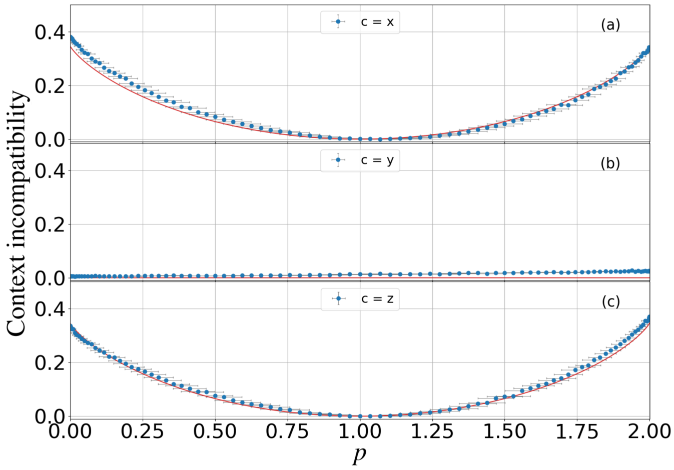

Fig. 2

Theoretical prediction of the context incompatibility $\mathcal{I}_{\mathbb{C}}$ (red line) and experimental results (blue dots) for the contexts (a) $\mathbb{C} = \{\rho_x, \sigma_x, \sigma_z\}$, (b) $\mathbb{C} = \{\rho_y, \sigma_x, \sigma_z\}$ and (c) $\mathbb{C} = \{\rho_z, \sigma_x, \sigma_z\}$ as a function of p. The calculations were performed using base-e logarithms. The error bars, on the order of 10^{-2}, are smaller than the data points.

The results show excellent agreement with theory. Compatibility ($\mathcal{I}_{\mathbb{C}} = 0$) is observed for the maximally mixed state ($p = 1$) and for states on the y-axis (orthogonal to both measurement bases). In contrast, for states aligned with the measurement axes (x or z), incompatibility is clearly non-zero and peaks for pure states ($p = 0, 2$), where the non-commutativity of the observables is most impactful. A second experiment, shown in Fig. 3, investigated incompatibility as a function of the observables. We fixed the state along the z-axis and $B = \sigma_z$, while varying A from σ_z to σ_x. The results confirm that incompatibility vanishes when observables commute and peaks when they are mutually unbiased. For the maximally mixed state, the context is always compatible, regardless of the observables.

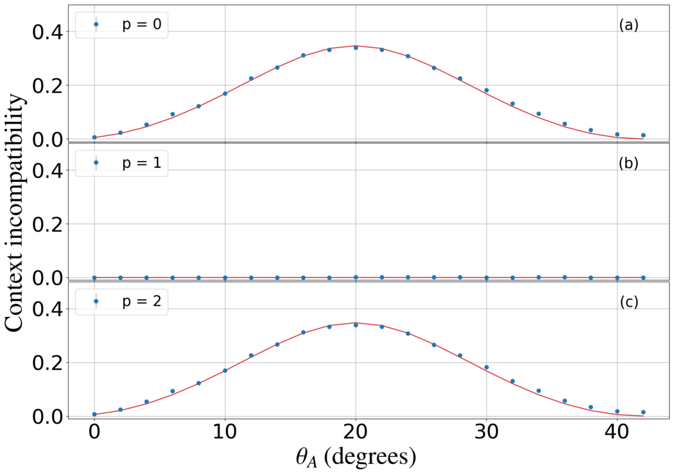

Fig. 3

Context incompatibility $\mathcal{I}_{\mathbb{C}}$ for the context $\mathbb{C} = \{\rho_z, A, \sigma_z\}$ as a function of the wave plate angle θ_A that defines observable A. The panels correspond to (a) $p = 0$ (pure state), (b) $p = 1$ (maximally mixed state), and (c) $p = 2$ (orthogonal pure state). The red line is theory, blue dots are experimental data.

V. Conclusions

In this paper, we have presented the detailed experimental implementation and verification of a theory-independent context incompatibility criterion. Focusing on the experimental methodology, we have detailed how arbitrary single-photon qubit mixed quantum states can be precisely prepared and how sequential non-selective measurements can be realized using a quantum optics apparatus that employs the optical path as an auxiliary degree of freedom. Our experimental results unequivocally demonstrate that nature violates the principle of context compatibility, and the degree of this violation is in remarkable agreement with the quantum predictions derived from the TICI formalism [37]. The dependence of incompatibility on both the system's state and the commutation relation of the observables was clearly confirmed. This work, therefore, provides a robust experimental validation of a notion of incompatibility that generalizes previous formulations, such as Heisenberg's uncertainty principle. Our results pave the way for future experimental investigations of context incompatibility in higher-dimensional and multipartite systems, where its implications for quantum information tasks and foundational questions can be further explored.

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