Quantum State Consistency under LOCC: Reconciling Non-Commutative Operations via Classical Coordination

Andresso da Silva and Francisco M. de Assis

***Abstract*— We investigate the problem of maintaining consistency between quantum states held by two spatially separated parties, Alice and Bob, under a set of local unitary operations $\mathcal{U} = \{U_1, \dots, U_n\}$ and classical communication (LOCC). For sequential protocols, we show that identical states can always be preserved, as operations are applied in the same order by both parties. In contrast, simultaneous operations introduce challenges due to the non-commutative nature of quantum operators: commutation relations determine whether consistency can be preserved directly, and when operations do not commute, corrective protocols are required. We illustrate these phenomena with explicit examples, including block-based corrections.**

***Keywords*— Commutativity, Quantum States, LOCC.**

I. Introduction

Data consistency is a central problem in classical computer science. In distributed systems, such as replicated databases, different parties may perform concurrent operations, and it is essential to ensure that all replicas remain coherent regardless of the order in which local updates are applied. Classical protocols, including two-phase commit [1] and consensus algorithms [2], have been developed to address this challenge.

In the quantum domain, however, maintaining consistency becomes substantially more intricate. Unlike classical data, quantum states cannot be perfectly cloned due to the no-cloning theorem [3], which imposes fundamental restrictions on copying arbitrary states. Additionally, the local unitary operations available to each party may not commute, meaning that the order of operation application can irreversibly alter the resulting state. These unique features make the problem of quantum state consistency fundamentally different from classical concurrency.

In this work, we consider two distant parties, Alice and Bob, each holding copies of the same quantum state $|\psi\rangle$ and sharing a set of local unitary operations $\mathcal{U} = \{U_1, \dots, U_n\}$. The parties can coordinate only through bidirectional classical communication (LOCC). We investigate how to preserve the consistency of their states under both sequential and simultaneous application of unitary operations. Our approach leverages reconciliation protocols based on unitary corrections applied after the indices of operations are communicated, ensuring that the states remain identical when possible.

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The motivation for our approach is directly related to the fundamental challenge of scalability in quantum computing. The construction of large processors can be achieved through the interconnection of smaller devices, which requires the efficient transfer of quantum information between different nodes [4]. In this scenario, by focusing on unitary operations and transformations via LOCC, we establish a solid theoretical framework to analyze and characterize the necessary conditions for manipulating states in distributed systems.

The remainder of the paper is organized as follows. In Section II, we introduce the formal framework, including the definitions of the shared quantum states, the set of local unitary operations, and the classical communication model (LOCC). Section III presents the main results, covering both sequential and simultaneous application of unitary operations, the role of commutation relations, reconciliation protocols, and illustrative examples using commutation graphs. Finally, Section IV summarizes the key findings, discusses the fundamental limitations, and outlines potential directions for future research.

II. Fundamentals

A. Quantum Operators and States

The Hilbert space is denoted by $\mathcal{H}$. An operator A acts on a state $|\psi\rangle \in \mathcal{H}$ and maps it to another state $|\phi\rangle \in \mathcal{H}$, i.e., $A|\psi\rangle = |\phi\rangle$. The set of all linear operators acting on $\mathcal{H}$ is denoted by $\mathcal{B}(\mathcal{H})$. A unitary operator U is one that satisfies $UU^\dagger = I$, where $U^\dagger$ is the conjugate transpose of U and I is the identity operator. Two operators A and B are said to be compatible if they commute, that is, if $[A, B] = AB - BA = 0$. Compatibility implies that the order of applying A and B does not affect the resulting state, which is a key property in designing protocols for maintaining consistency between distributed quantum states.

B. LOCC and State Transformations

Local Operations and Classical Communication (LOCC) forms a fundamental framework in quantum information theory, describing scenarios in which spatially separated parties manipulate their respective subsystems using local quantum operations while coordinating through classical communication channels [5], [6]. LOCC protocols are widely used in tasks such as quantum teleportation, entanglement distillation, and quantum state discrimination, as they respect the physical constraints of spatial separation and prohibit instantaneous transfer of quantum information [7], [8], [9]. In the context of maintaining consistency between distributed quantum states, LOCC provides an operational paradigm under which parties can reconcile differences in their local states by exchanging classical information about the operations applied, without violating the principles of quantum mechanics.

The conversion of quantum states into other states is closely related to LOCC operations and the concept of majorization between states. To understand this relationship, it is necessary to introduce some definitions and results, which can be found in works such as [10], [6], [11], [12].

Definition 1 (Vector in descending order): Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be a vector. It is said to be in decreasing order when its elements are arranged such that

$$\mathbf{x}^{\downarrow} = \left(x_{(1)}, x_{(2)}, \ldots, x_{(n)}\right), \tag{1}$$

where $x_{(1)} \geq x_{(2)} \geq \cdots \geq x_{(n)}$.

Using Definition 1, one can define the majorization relation between two vectors.

Definition 2 (Majoration): A vector $\mathbf{y} \in \mathbb{R}^n$ is said to majorize another vector $\mathbf{x} \in \mathbb{R}^n$, denoted by $\mathbf{x} \prec \mathbf{y}$, if and only if

$$\sum_{i=1}^{k} x_i^{\downarrow} \leq \sum_{i=1}^{k} y_i^{\downarrow} \tag{2}$$

for all $k = 1, \ldots, n-1$ and equality holds for $k = n$ (i.e., the vectors are normalized). Moreover, $x_i^{\downarrow}$ and $y_i^{\downarrow}$ denote the i-th elements of $\mathbf{x}^{\downarrow}$ and $\mathbf{y}^{\downarrow}$, respectively.

It is worth noting that majorization defines a partial order on the set of vectors, and it always holds that a vector majorizes itself, i.e., $\mathbf{x} \prec \mathbf{x}$.

Example 1: As an example, consider $\mathbf{x} = (1/3, 1/3, 1/3)$ and $\mathbf{y} = (0, 0, 1)$. Thus, $\mathbf{x}^{\downarrow} = (1/3, 1/3, 1/3)$ e $\mathbf{y}^{\downarrow} = (1, 0, 0)$.

For $k = 1$, we have that

$$x_1^{\downarrow} \leq y_1^{\downarrow} \Leftrightarrow 1/3 \leq 1. \tag{3}$$

For the case $k = 2$, we have

$$x_1^{\downarrow} + x_2^{\downarrow} \leq y_1^{\downarrow} + y_2^{\downarrow} \Leftrightarrow 1/3 + 1/3 \leq 1. \tag{4}$$

Finally, for $k = 3$, we have

$$x_1^{\downarrow} + x_2^{\downarrow} + x_3^{\downarrow} \leq y_1^{\downarrow} + y_2^{\downarrow} + y_3^{\downarrow} \Leftrightarrow 1/3 + 1/3 + 1/3 = 1. \tag{5}$$

Since the conditions of Definition 2 are satisfied, $\mathbf{y}$ majorizes $\mathbf{x}$, $\mathbf{x} \prec \mathbf{y}$.

Nielsen's theorem [10] describes the conditions under which one bipartite pure quantum state can be converted into another using LOCC.

Theorem 1 ([10]): Let $|\psi\rangle$ be a state shared by Alice and Bob, and let $\rho_\psi = \mathrm{tr}_B(|\psi\rangle\langle\psi|)$ denote Alice's reduced state. Let λ_ψ be the vector of eigenvalues of ρ_ψ. Then, the state $|\psi\rangle$ can be transformed into $|\phi\rangle$ via LOCC, denoted $|\psi\rangle \to |\phi\rangle$, if and only if $\lambda_\psi \prec \lambda_\phi$.

Example 2: Let

$$|\psi\rangle = \sqrt{0.8}\,|00\rangle + \sqrt{0.2}\,|11\rangle \tag{6}$$

the state shared by Alice and Bob. Let us also consider the Hadamard matrix, defined as

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{7}$$

The Hadamard matrix is unitary. If Alice applies H to her part of the state, then

$$H \otimes I = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \tag{8}$$

and $|\phi\rangle = (H \otimes I)\,|\psi\rangle$, where

$$(H \otimes I)\,|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} \sqrt{0.8} \\ 0 \\ 0 \\ \sqrt{0.2} \end{bmatrix} \tag{9}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} \sqrt{0.8} \\ \sqrt{0.2} \\ \sqrt{0.8} \\ -\sqrt{0.2} \end{bmatrix} \tag{10}$$

$$|\phi\rangle = \frac{1}{\sqrt{2}}(\sqrt{0.8}(|00\rangle + |10\rangle) + \sqrt{0.2}(|01\rangle - |11\rangle)). \tag{11}$$

Thus, since $\lambda_\psi = (0.8, 0.2)$ and $\lambda_\phi = (0.8, 0.2)$, the Hadamard operation preserves the eigenvalues, and we have $\lambda_\psi \prec \lambda_\phi$.

In general, unitary matrices do not change the eigenvalues of a density matrix. In this work, we focus exclusively on unitary operations, which represent a special case in state conversion. However, the framework can be extended to include more general quantum operations, which allows for the conversion of quantum states via LOCC protocols. This is particularly relevant when considering transformations that go beyond simple unitary evolution, such as those involving measurements or other completely positive trace-preserving maps, where the majorization conditions play a crucial role in determining the feasibility of state conversion.

III. Quantum State Consistency

Suppose two parties, Alice and Bob, prepare the same states $|\psi_A\rangle$ and $|\psi_B\rangle$, such that $|\psi_A\rangle = |\psi_B\rangle = |\psi\rangle$, and also share a set $\mathcal{U} = U_1, U_2, \ldots, U_n$ of local unitary operations. Approximate copies of the state can be obtained through entanglement dilution [5]. Alice and Bob are spatially separated but can communicate through a bidirectional (duplex) classical channel. Whenever one party applies an operation U_j, they can send the corresponding index j so that the other party can apply the same operation to their state. The goal is to maintain the two prepared states identical, using only local operations and classical communication (LOCC).

This problem can be interpreted as a quantum analogue of concurrency in classical databases, where operations may be executed by different parties and it is necessary to ensure that the states remain consistent across all databases. The quantum version introduces a key distinction: in general, it is not possible to perfectly copy a state from one party to another, as in the classical case. Therefore, an alternative mechanism is required to maintain consistency.

The initially considered scenario is sequential. For $|\psi_A\rangle^{(0)} = |\psi_B\rangle^{(0)} = |\psi\rangle$, where the superscript denotes the initial time $t = 0$, if Alice performs the operation U_{j_1} and sends j_1 to Bob, the states are updated as follows:

$$|\psi_A\rangle^{(1)} = U_{j_1} |\psi\rangle \tag{12}$$

and

$$|\psi_B\rangle^{(0)} = |\psi\rangle \xrightarrow{j_1} |\psi_B\rangle^{(1)} = U_{j_1} |\psi\rangle, \tag{13}$$

where $\xrightarrow{j_1}$ indicates that the index j_1 has been received and the operation U_{j_1} has been applied to the current state. After these operations, the equality $|\psi_A\rangle^{(1)} = |\psi_B\rangle^{(1)}$ still holds.

If Bob then performs the operation U_{j_2} and sends j_2 to Alice, the states will be given by

$$|\psi_B\rangle^{(2)} = U_{j_2} U_{j_1} |\psi\rangle \tag{14}$$

and

$$|\psi_A\rangle^{(1)} = U_{j_1} |\psi\rangle \xrightarrow{j_2} |\psi_B\rangle^{(2)} = U_{j_2} U_{j_1} |\psi\rangle. \tag{15}$$

Once again, the states remain equal, $|\psi_A\rangle^{(2)} = |\psi_B\rangle^{(2)}$. The process described is represented by the diagram in Fig. 1.

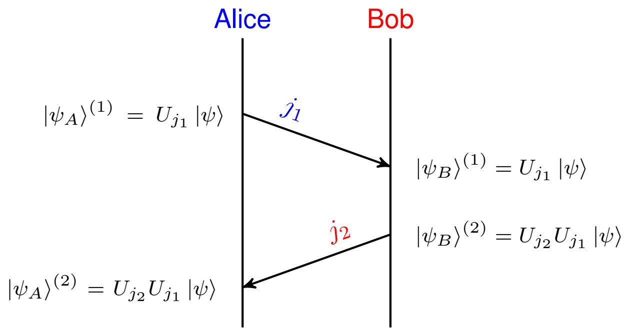

Fig. 1. Application of the operators $U_i \in \mathcal{U}$ in a sequential manner. Alice applies the operator U_{j_1} and sends the index j_1 to Bob, who waits for its reception before applying the same operation. Bob then applies the operator U_{j_2} and sends the index j_2 to Alice, who waits for its reception before applying any further operations.

It can be observed that, for sequential processes like those described, it is always possible to ensure that $|\psi_A\rangle^{(t)} = |\psi_B\rangle^{(t)}$ when the set of operations consists of unitary operations. This is because, if Alice and Bob follow the protocol, the operations are executed in the same order by both parties.

Suppose that Alice and Bob can apply operations with indices j and k, respectively. In this way, they do not need to wait to receive the other party's index before applying the operators to their copies. After applying their operations, each party sends the corresponding index through the classical channel. If U_j and U_k commute (i.e., are compatible), Alice and Bob can maintain identical states, since $U_j U_k = U_k U_j$. The following procedures describe how to handle cases where the operators do not commute, in order to preserve consistency of the state copies.

Since both parties possess the set of operators $\mathcal{U}$, they can determine in advance which operations are compatible and which are incompatible. Upon receiving the index of the other party's operation, they can identify whether two incompatible operations have been applied. When incompatible operations are detected, it is necessary to correct the sequence locally.

Considering the scenario of simultaneity (e.g., where each party applies an operation before receiving the index from the other), the resulting states will be

$$|\psi_A\rangle^{(1)} = U_{j_1} |\psi\rangle \xrightarrow{j_2} |\psi_A\rangle^{(2)} = U_{j_2} U_{j_1} |\psi\rangle \tag{16}$$

and

$$|\psi_B\rangle^{(1)} = U_{j_2} |\psi\rangle \xrightarrow{j_1} |\psi_B\rangle^{(2)} = U_{j_1} U_{j_2} |\psi\rangle. \tag{17}$$

This case is illustrated by the diagram in Fig. 2.

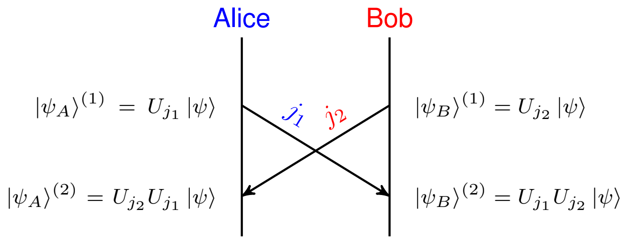

Fig. 2. Application of the operators $U_i \in \mathcal{U}$ in a simultaneous manner. Neither Alice nor Bob needs to wait for the reception of the index corresponding to the operation performed by the other party before applying a local operation.

In this case, $|\psi_A\rangle^{(2)} = |\psi_B\rangle^{(2)}$ if and only if $U_{j_1} U_{j_2} = U_{j_2} U_{j_1}$, or equivalently, if $[U_{j_1}, U_{j_2}] = 0$. If $U_{j_1} U_{j_2} \neq U_{j_2} U_{j_1}$, it becomes necessary to restore $|\psi_A\rangle^{(t)} = |\psi_B\rangle^{(t)}$ using LOCC.

To address this problem, we consider the state of each party after the simultaneous operations:

$$|\psi_A\rangle^{(2)} = U_{j_2} U_{j_1} |\psi\rangle \tag{18}$$

and

$$|\psi_B\rangle^{(2)} = U_{j_1} U_{j_2} |\psi\rangle. \tag{19}$$

One way to recover the same state in both parties involves transforming $U_{j_1} U_{j_2}$ into $U_{j_2} U_{j_1}$ in some manner. Since the operators in $\mathcal{U}$ are unitary, this transformation can be achieved using the protocol described below. Alice retains her state, $|\psi_A\rangle^{(2')} = U_{j_2} U_{j_1} |\psi\rangle$, while Bob proceeds as follows upon detecting that Alice has sent an index j_1 corresponding to an operation that is incompatible with j_2:

$$|\psi_B\rangle^{(2)} = U_{j_1} U_{j_2} |\psi\rangle \tag{20}$$

$$|\psi_B\rangle^{(2')} = (U_{j_2} U_{j_1} U_{j_2}^\dagger U_{j_1}^\dagger) U_{j_1} U_{j_2} |\psi\rangle = U_{j_2} U_{j_1} |\psi\rangle, \tag{21}$$

thus allowing Bob to reach $|\psi_B\rangle = U_{j_2} U_{j_1} |\psi\rangle$. It is worth noting that it was necessary to undo the application of $U_{j_1} U_{j_2}$, returning $|\psi_B\rangle$ to the initial state, and then reapply the operations U_{j_1} and U_{j_2} in the same order as applied by Alice.

The process would be simpler if, before applying the operation corresponding to Alice's j_1 on his state, Bob checked whether U_{j_1} commutes with U_{j_2}. If the operations do not commute, Bob could then follow the procedure

$$|\psi_B\rangle^{(1)} = U_{j_2} |\psi\rangle \tag{22}$$

$$|\psi_B\rangle^{(2)} = U_{j_2} U_{j_1} U_{j_2}^\dagger U_{j_2} |\psi\rangle = U_{j_2} U_{j_1} |\psi\rangle. \tag{23}$$

That is, Bob would only need to undo the application of U_{j_2} and then apply U_{j_1} followed by U_{j_2}, in the same order as Alice applied them.

This protocol can be generalized to the case where each party applies more than one operation and sends all the corresponding indices. The commutation relationships between operators can be represented using a graph in which the vertices correspond to the operators, and edges connect pairs of operators that commute. For example, consider the set of operations $\mathcal{U} = \{U_1, \ldots, U_5\}$, which commute according to the commutation/compatibility graph shown in Fig. 3.

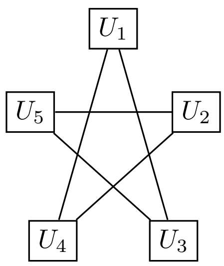

Fig. 3. Compatibility graph between operations, where each vertex represents an operator and an edge connecting two vertices indicates that the corresponding operators commute (i.e., they are compatible).

Fig. 4 shows the diagram corresponding to the case where Alice and Bob perform the operations $U_1U_2U_4$ and $U_1U_3U_5U_2$, respectively, on their states $|\psi_A\rangle$ and $|\psi_B\rangle$.

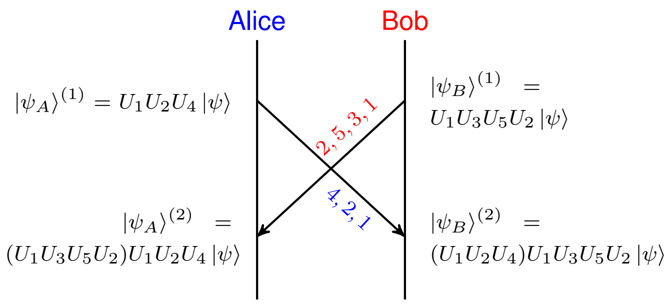

Fig. 4. Alice and Bob can apply multiple operations on their respective copies of the quantum state before sending the corresponding indices to each other.

In this case, the goal is to determine which operations Bob must perform to restore $|\psi_A\rangle^{(2)} = |\psi_B\rangle^{(2)}$. Using the commutation/compatibility relationships provided by the graph in Fig. 3, one can infer equivalent sequences of operations performed by Alice and Bob. Equivalent sequences are obtained by swapping the positions of operations that commute and are contiguous within the sequence. Thus, we have

- Alice: $U_1U_2U_4 \equiv U_1U_4U_2 \equiv U_4U_1U_2$ e
- Bob: $U_1U_3U_5U_2 \equiv U_1U_5U_3U_2 \equiv U_3U_1U_5U_2 \equiv U_3U_1U_2U_5 \equiv U_3U_1U_2U_5$.

Following the protocol, Alice retains her state $|\psi_A\rangle^{(2)} = (U_1U_3U_5U_2)(U_1U_2U_4)|\psi\rangle$, while Bob must apply operations to transform $(U_1U_2U_4)(U_1U_3U_5U_2)$ into $(U_1U_3U_5U_2)U_1U_2U_4$. Using the compatibility relationships, Bob can determine that the operation sequences are not equivalent, and it becomes necessary to apply the inverses of certain operations to maintain consistency between the states.

In this way, upon receiving the indices $2, 5, 3, 1$ from Alice, Bob can apply the following operations:

$$|\psi_B\rangle^{(1)} = U_1U_3U_5U_2 \tag{24}$$

$$|\psi_B\rangle^{(2)} = (U_1U_3U_5U_2)(U_1U_2U_4)(U_1U_3U_5U_2)^{\dagger}U_1U_3U_5U_2, \tag{25}$$

ensuring that $|\psi_A\rangle^{(2)} = |\psi_B\rangle^{(2)}$. It is worth noting that the commutation relationships can be used to reduce the number of operations performed. The minimum number of operations in this case can be determined using an edit distance that takes commutation relations into account [13], [14].

The previous examples share a common feature. The operations of Alice and Bob correspond to distinct blocks of the form U_BU_A and U_AU_B, respectively, where $U_A = U_{i_1} \cdots U_{i_l}$, with $U_i \in \mathcal{U}$, and $U_B = U_{j_1} \cdots U_{j_k}$, with $U_j \in \mathcal{U}$. For the previous example, $U_A = U_1U_2U_4$ and $U_B = U_1U_3U_5U_2$. In this way, if Alice maintains the sequence of operations U_BU_A, it is always possible for Bob to obtain the sequence $U_BU_AU_B^{\dagger}U_B = U_BU_A$, which preserves consistency.

In the following, we analyze a more general example. The Fig. 5 shows the interactions between Alice and Bob. Only the sequence of operations is depicted in the figure to keep the notation less cluttered. The commutation relations between the operations are given by the graph in Fig. 3.

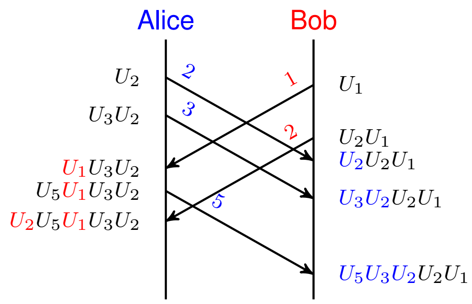

Fig. 5. Application of multiple operations in a simultaneous manner. The initial states have been omitted from the figure, only the sequences of operations are shown.

The final state for Alice is $|\psi_A\rangle^{(5)} = U_2U_5U_1U_3U_2|\psi\rangle$, while for Bob it is $|\psi_B\rangle^{(5)} = U_5U_3U_2U_2U_1|\psi\rangle$. It can be noted that by applying Alice's operations directly, Bob's state effectively becomes $|\psi_B\rangle^{(5)} = U_5U_3U_1|\psi\rangle$. Once again, the goal is to determine whether it is possible to obtain $|\psi_A\rangle^{(5)}$ from $|\psi_B\rangle^{(5)}$ and the indices sent by Alice.

Bob's sequence, $U_5U_3U_2U_2U_1$, can be decomposed into blocks U_AU_B, corresponding to the operations performed by Alice and Bob, where $U_A = U_5U_3U_2$ and $U_B = U_2U_1$. However, the same does not hold for Alice. Previously, it was possible to recover Alice's sequence by applying $U_BU_AU_B^{\dagger}U_B$. In this case, Bob's new sequence is given by $U_BU_AU_B^{\dagger}U_B = U_2U_1U_5U_3U_2$. It is easy to verify that this sequence is not equivalent to the one generating the state $|\psi_A\rangle^{(5)} = U_2U_5U_1U_3U_2|\psi\rangle$, since U_1 and U_5 do not commute and are applied in different orders in the two sequences. Therefore, even if Bob's sequence can be decomposed into blocks, it is also necessary that the same is possible for Alice.

In general, it is not always possible to guarantee consistency between Alice's and Bob's states based on the methods presented so far. Each party knows the order in which they applied operations on their own copy of the state, as well as the order in which the other party applied operations on their copy. However, the ordering relationship between locally applied operations and those applied by the other party is unknown. Therefore, if any incompatible operation is applied before the sequence is updated, inconsistencies may arise, as was the case in the previous example.

For the previous example, in Alice's frame of reference, she first applied U_1 received from Bob, then applied U_5 and sent it to Bob. From Bob's perspective, he applied U_1 and subsequently received from Alice the index 5 corresponding to U_5. It is worth noting that the inconsistency would occur even if Bob applied the block-based correction after each new index received from Alice.

In summary, the analysis shows that while sequential application of unitary operations allows Alice and Bob to maintain identical states using LOCC, the situation becomes more intricate under simultaneous operation scenarios. Commutation relations between operators play a central role in determining whether consistency can be preserved. When operations commute, simple protocols suffice to restore equality between the states. However, when operations do not commute, additional corrections are required, and even then, global consistency is not always guaranteed. These findings highlight the fundamental difference between classical and quantum concurrency, emphasizing the limitations imposed by the non-commutative nature of quantum operations.

IV. Conclusions

In this work, we analyzed the problem of maintaining consistency between quantum states held by two distant parties, Alice and Bob, under the application of local unitary operations and classical communication (LOCC). We showed that for sequential processes, the states can always be kept identical, as both parties can follow the same operation order. In scenarios involving simultaneous operations, the situation becomes more complex: the commutation relations between operators determine whether consistency can be preserved directly, and when operations do not commute, additional corrective protocols are required. We also highlighted that even with block-based corrections, global consistency is not always guaranteed, illustrating a fundamental distinction between classical and quantum concurrency.

Future research could focus on extending these results to larger multipartite systems and investigating efficient algorithms to compute minimal corrective sequences using commutation graphs. Another promising direction is the exploration of approximate protocols where perfect consistency is not achievable, but the resulting states remain within an acceptable fidelity threshold. Finally, integrating these protocols into practical quantum networks and distributed quantum computing platforms could provide insights into real-world limitations and optimization strategies for quantum concurrency management.

Acknowledgments

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