A Ramsey-Guaranteed Architecture for Robust Multi-Party Quantum Protocols

Luís Guilherme Miranda Spengler and João Vítor Batista Ferreira

***Abstract*— Multi-party quantum protocols require robust network topologies, a challenge in realistic networks with variable link fidelity. We propose an architecture where Ramsey's Theorem guarantees the existence of functional substructures. In a 6-node network, a three-node cluster connected by links of similar fidelity (either high or medium) is mathematically assured. We demonstrate the utility of this guaranteed framework by applying it to cyclic quantum teleportation. This approach offers a novel method for certifying network capabilities, ensuring reliable protocol deployment even in non-ideal conditions.**

***Keywords*— Ramsey's Theorem, Quantum Networks, Quantum Protocols.**

I. Introduction

Distributed Quantum Computing (DQC) has emerged as a critical field to overcome the limitations of individual quantum processors and to handle data that is inherently spread across different locations [1]. The success of DQC and other multi-party protocols, such as quantum secret sharing [2], often relies on linking multiple quantum nodes in specific ways to collaboratively execute complex algorithms.

However, establishing and maintaining high-quality network links is challenging in real-world scenarios. Factors such as environmental noise, resource limitations, or infrastructure availability cause the fidelity of entangled links to vary significantly across a network [3], [4], [5]. This non-uniform connection quality introduces an element of randomness and presents a major challenge for reliable protocol implementation.

This paper proposes a novel approach to quantum network design that leverages Ramsey's Theorem [6] to ensure the existence of specific, functional substructures. We introduce a network model based on shared entangled pairs and demonstrate that a guaranteed three-party, fully-connected subsystem will always emerge in a six-party network. This substructure provides a robust foundation for various quantum protocols, and we will use cyclic quantum teleportation [7] as a primary example to demonstrate the architecture's utility and strategic advantages.

II. The Ramsey Network Model

Our proposed network consists of R individuals, or nodes. The core principles of the model are as follows:

- **Entangled Pairs**: Every two individuals in the network share a pair of entangled particles.
- **Quantum Channels**: The establishment of these entangled pairs occurs through a medium known as a quantum channel. We assume all connections are attempted through the same type of quantum channel. However, due to factors like environmental noise and distance, the resulting fidelity of each entangled pair varies. We categorize these usable links into two tiers: 1. The high-fidelity ($F \geq 0.9$) "Purple" link. 2. The medium-fidelity ($0.6 \leq F < 0.9$) "Green" link, where fidelity is sufficient for operations but requires more error correction. Any connection with a fidelity below 0.6 is considered a failed or invalid link and is not included in our graph. The network is thus represented as a graph, where the edges are colored "Purple" or "Green" based on their measured fidelity.
- **Graph Theory Representation**: This network can be visualized using graph theory, where individuals are represented as vertices and the connections established by the quantum channels are represented as edges. Each edge is color-coded "Purple" or "Green" according to its corresponding channel.

For our primary example, we consider a network of $R = 6$ people, as depicted in Figure 1.

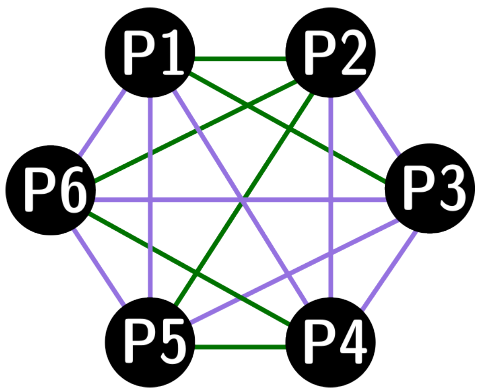

Fig. 1. A group of $R = 6$ individuals (nodes) connected via 15 quantum channels (edges) where every two people share a pair of entangled particles using either the "Purple" or "Green" channel.

Luís Guilherme Miranda Spengler, Instituto de Física, Universidade Federal de Mato Grosso do Sul, Campo Grande-MS, e-mail: luis.spengler@ufms.br; João Vítor Batista Ferreira, Instituto de Física, Universidade Federal de Mato Grosso do Sul, Campo Grande-MS, e-mail: joao.ferreira@ufms.br.

III. Results: The Guaranteed Monochromatic Substructure

By applying Ramsey's Theorem to the proposed 6-node network, we can guarantee a specific structural outcome. The theorem states that for a group of $R = 6$ individuals connected in this manner, there will always exist a subset of $k = 3$

individuals who all share an entangled pair with each other through the same quantum channel.

In the graph representation, this result means that no matter how the edges are colored "Purple" or "Green", there will always be at least one monochromatic triangle, which implies that within any 6-node network, there is always a cluster of three nodes connected by links of similar quality. This means we can be certain of finding either:

- A high-fidelity "Purple" triangle, ideal for sensitive computations.
- A medium-fidelity "Green" triangle, which can serve as a robust, guaranteed fallback.

IV. Application: Cyclic Quantum Teleportation

Any monochromatic triangle found within the network can serve as a framework for implementing multi-party protocols, such as cyclic quantum teleportation. REF[19].

The protocol proceeds as follows:

- **Setup:** The three vertices of a monochromatic triangle are designated as "Alice," "Bob," and "Charlie". Alice possesses an initial quantum state $|\psi\rangle_A$ to be teleported. The edges of the triangle represent the shared entangled pairs: $|\phi_+\rangle_{AB}$, $|\phi_+\rangle_{BC}$, and $|\phi_+\rangle_{CA}$.
- **Alice to Bob:** Alice performs a Bell-state measurement on her state $|\psi\rangle_A$ and her part of the entangled pair $|\phi_+\rangle_{AB}$. She sends the two-bit classical result to Bob, who applies a corresponding unitary operation to reconstruct the state $|\psi\rangle_A$. This step consumes the entangled pair between them.
- **Bob to Charlie:** Bob repeats the process. He performs a Bell-state measurement on the state he now holds and his part of the $|\phi_+\rangle_{BC}$ pair, sending the classical result to Charlie for reconstruction. The $|\phi_+\rangle_{BC}$ pair is consumed.
- **Charlie to Alice:** Charlie performs a final Bell-state measurement with the $|\phi_+\rangle_{CA}$ pair and sends his result to Alice. Alice applies the final operation to reconstruct her original state, completing the cycle. The last entangled pair is consumed in this step.

The discovery of a monochromatic triangle allows for strategic protocol deployment. For a high-priority task, one would preferentially use a "Purple" triangle to minimize errors and resource cost for correction. If only a "Green" triangle is available, the protocol can still be executed, but with the expectation of applying more rigorous error mitigation techniques. Ramsey's Theorem gives us the confidence that at least one of these options will be available.

V. Conclusion

We have proposed a quantum network architecture that uses Ramsey's Theorem to find guaranteed order within the inherent variability of a single channel's fidelity. Our model shows that in a 6-node network where connections are classified as either high- or medium-fidelity, the existence of a monochromatic triangle is a mathematical certainty.

This result has significant practical implications. It transforms the challenge of random link quality into a predictable advantage. The guaranteed presence of a three-node subsystem connected by links of similar fidelity—either high-quality for sensitive tasks or medium-quality as a robust fallback—provides a concrete assurance for network operators. Our analysis of cyclic quantum teleportation serves as a powerful proof-of-concept for this model, allowing for strategic deployment of multi-party protocols to enhance network reliability and efficiency.

The Ramsey-based framework is a versatile tool applicable to other critical protocols, including quantum secret sharing and distributed quantum computing. Ultimately, this approach offers a novel method for certifying the functional capabilities of quantum networks, ensuring that even in the face of real-world imperfections, robust substructures for computation and communication can always be found.

References

[1] Juan C. Boschero et al. "Distributed Quantum Computing: Applications and Challenges". In: *Intelligent Computing*. Ed. by Kohei Arai. Cham: Springer Nature Switzerland, 2025, pp. 100–116. ISBN: 978-3-031-92602-0.

[2] Mark Hillery, Vladimír Bu žek, and André Berthiaume. "Quantum secret sharing". In: *Phys. Rev. A* 59 (3 Mar. 1999), pp. 1829–1834. DOI: 10.1103/PhysRevA.59.1829. URL: https://link.aps.org/doi/10.1103/PhysRevA.59.1829.

[3] Ronald E. Meyers. "Free-Space and Atmospheric Quantum Communications". In: *Advanced Free Space Optics (FSO)*. Springer New York, Sept. 2014, pp. 343–387. ISBN: 9781493909186. DOI: 10.1007/978-1-4939-0918-6_10. URL: http://dx.doi.org/10.1007/978-1-4939-0918-6_10.

[4] Yihui Quek et al. "Exponentially tighter bounds on limitations of quantum error mitigation". In: *Nature Physics* 20.10 (July 2024), pp. 1648–1658. ISSN: 1745-2481. DOI: 10.1038/s41567-024-02536-7. URL: http://dx.doi.org/10.1038/s41567-024-02536-7.

[5] Xinyu Chen et al. "High-fidelity, low-latency polarization quantum state transmissions over a hollow-core conjoined-tube fiber at around 800nm". In: *Photonics Research* 9.4 (Mar. 2021), p. 460. ISSN: 2327-9125. DOI: 10.1364/prj.409521. URL: http://dx.doi.org/10.1364/PRJ.409521.

[6] F. P. Ramsey. "On a Problem of Formal Logic". In: *Proceedings of the London Mathematical Society* s2-30.1 (1930), pp. 264–286. ISSN: 0024-6115. DOI: 10.1112/plms/s2-30.1.264. URL: http://dx.doi.org/10.1112/plms/s2-30.1.264.

[7] Ying-Xuan Chen et al. "Cyclic quantum teleportation". In: *Quantum Information Processing* 16.8 (July 2017). ISSN: 1573-1332. DOI: 10.1007/s11128-017-1648-1. URL: http://dx.doi.org/10.1007/s11128-017-1648-1.