Realism-Based Approach To Afshar's Experiment Paradox

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***Abstract*— This study reinterprets Afshar's experiment, criticized for allegedly violating Bohr's principle of complementarity, without relying on Yasin and Greenberger's complementarity relation. Using the Bilobran–Angelo reality criterion and quantum irrealism quantification, it analyzes the analogy with Unruh's experiment (Mach–Zehnder interferometer). It shows that wave-like and particle-like behaviors always satisfy an inequality, thus preserving complementarity. The approach enables continuous monitoring of the quantum state, eliminating the need for retrodiction-based inferences.**

***Keywords*— Principle of Complementarity, Afshar Experiment, Realism.**

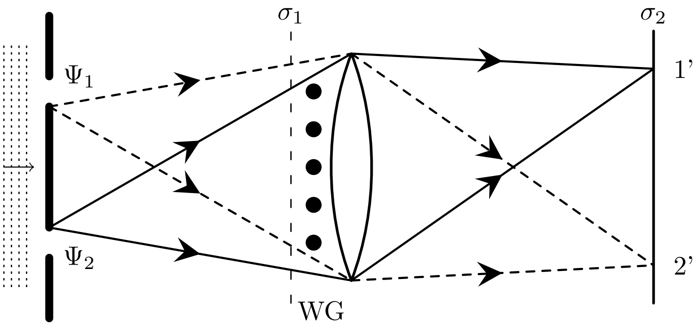
I. Afshar's Experiment

Using a thought experiment proposed by Einstein, and Shannon's entropy, Wooters and Zurek [1] enunciated Bohr's principle of complementarity for double slit (or "*wich-way*") experiments. This statement was equated by Greenberger and Yasin [2], an inequality called the complementarity relation, given by:

$$V^2 + K^2 \leq 1, \tag{1}$$

where V represents the visibility of the interference pattern, ranging from $0 \leq V \leq 1$. As for K, called information "which-way" information - or distinguishability - varies from $0 \leq K \leq 1$ and seeks to measure the information from which slit the quantum object originates; for $K = 1$ the information is total, while for $K = 0$ it is none.

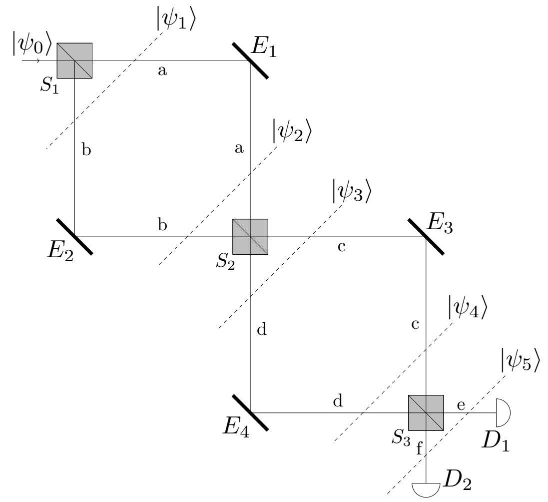
In 2005, an article was published by an Iranian physicist, Shahriar S. Afshar, claimed to have experimentally demonstrated the violation of Bohr's principle of complementarity [3]. Based on complementarity relation 1, Afshar proposes a modified double slit experiment, as shown in the following figure.



The luminous intensity of the σ_2 detectors is measured for three cases: (a) with both slits open and without the grating installed, (b) with one of the slits blocked and with the grating installed, and (c) with both slits open and with the grating installed. From (a) to (b) the total radiant flux is reduced by $\tilde{R} = (6.6 \pm 0.2)\%$, while from (a) to (c) the attenuation of the radiant flux is $R = (-0.1 \pm 0.2)\%$. With these results, Afshar finds that, due to the low attenuation in (c), it is certain that there is no diffraction caused by the grating, since there is no considerable attenuation in the detector. Therefore, he concludes that there is an interference pattern present in the grating sections, which gives a measure of wave behavior, such that $V \simeq 1$. However, due to the application of *retrodition*, it would be possible to ascertain the slit from which the respective photons originated and thus have $K \simeq 1$. He considered this to be a serious violation of the Yasin and Greenberger complementarity relation, since, in the relation of the equation 1, we have $V^2 + K^2 \simeq 2 > 1$. Which, according to him, meant that the Bohr's complementarity principle was violated.

II. Unruh's Experiment

The canadian physicist William Unruh [5] published on his website an analogy of the apparatus with an arrangement of Mach-Zehnder interferometers.



If we obstruct one of the paths a or b in the figure, detection will only be made on one detector. By the same arguments of *retrodiction*, it turns out that the path information would be total, so that $K = 1$ when both slits are open. In this case, there will be constructive interference in the path d and destructive interference in path c. As in Afshar's experiment, if we place an obstacle in the paths of destructive interference, it will play a role similar to that of the wire grid. It is reasonable to say that the visibility inferred by the obstacle is total, such that $V = 1$. Therefore, in the Yasin and Greenberger Complementarity relation, we will have that $V^2+K^2 = 2 > 1$. Qualitatively, this framework is the same as Afshar's, but with reduced dimensionality.

III. The Bilobran-Angelo Criterion of Reality

The BA criterion[7] defines that an observable $A = \sum_i a_i A_i$, with projectors $A_i = |a_i\rangle\langle a_i|$ acting on $\mathscr{H}_{\mathscr{A}}$, is real for a given preparation $\rho \in \bigotimes_{i=\mathscr{A}}^{\mathscr{N}} \mathscr{H}_i$ if, and only if,

$$\Phi_A(\rho) = \rho, \tag{2}$$

where $\Phi_A(\rho)$ is an unrevealed measurement map given by $\Phi_A(\rho) = \sum_i A_i \rho A_i$. Along these lines, Bilobran and Angelo suggest defining a quantifier that calculates this "entropic distance" for the BA criterion with von Neumann entropy. They state this quantifier as the *irreality* of an observable A, which is expressed as:

$$\mathfrak{I}_A(\rho) \equiv S(\Phi_A(\rho)) - S(\rho), \tag{3}$$

where A is the observable. It can be shown that the *irrealism* of any observable A is bounded as $0 \leq \mathfrak{I}_A(\rho) \leq \log d_{\mathscr{A}}$ [8], [9], where $d_{\mathscr{A}}$ is the dimension of $\mathscr{H}_{\mathscr{A}}$. Can also be defined a new quantifier, called *realism* $\mathfrak{R}_A(\rho)$, which acts as the dual correspondent of *irrealism*, where both its variations respect the following equality $\Delta\mathfrak{I}_A(\rho) + \Delta\mathfrak{R}_A(\rho) = 0$. From these, as well as from the previous limitation of irrealism, we can define the realism $\mathfrak{R}_A(\rho)$ of an observable A as

$$\mathfrak{R}_A(\rho) \equiv \log d_{\mathscr{A}} - \mathfrak{I}_A(\rho). \tag{4}$$

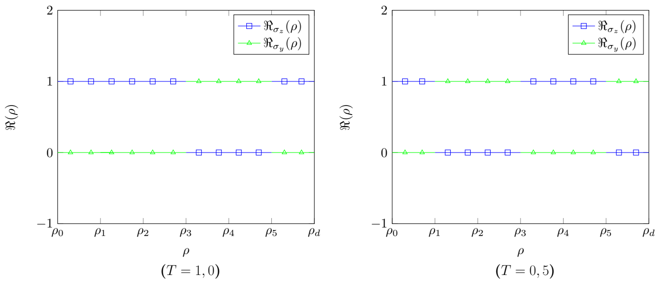
With these quantifiers, some interesting developments emerge. For example, it is possible to show that, for two incompatible observables A and A' acting on $\mathscr{H}_{\mathscr{A}}$ and applied in a ρ preparation of $\mathscr{H}_{\mathscr{A}} \otimes \mathscr{H}_{\mathscr{B}}$, when we have pure states, the realism for both observables respects the following inequality

$$\mathfrak{R}_A(\rho) + \mathfrak{R}_{A'}(\rho) \leq \log d_{\mathscr{A}} - E(\psi). \tag{5}$$

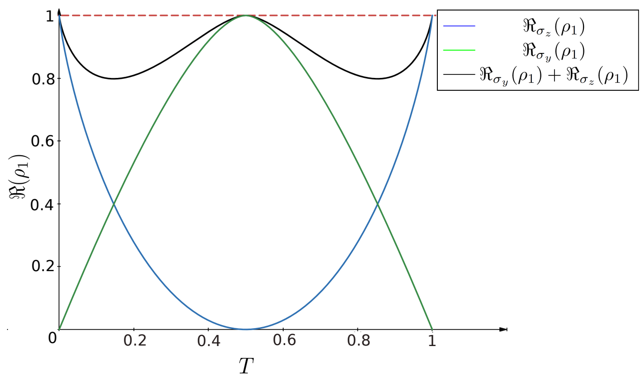
Where $E(\psi) = S(\rho_{\mathscr{A}(\mathscr{B})})$ is the entanglement entropy of the state. We defend that this inequality encapsulates Bohr's complementarity principle in a better way than Yasin and Greenberger's complementarity relation.

IV. Results & Conclusion

In the unruh experiment we can generalize the quantum states by $|\psi_1\rangle = \sqrt{T}\,|a\rangle + i\sqrt{R}\,|b\rangle\,; |\psi_2\rangle = \sqrt{R}\,|b\rangle + i\sqrt{T}\,|a\rangle\,; |\psi_3\rangle = \frac{\sqrt{R}-\sqrt{T}}{\sqrt{2}}\,|c\rangle + i\frac{\sqrt{R}+\sqrt{T}}{\sqrt{2}}\,|d\rangle\,; |\psi_4\rangle = \frac{\sqrt{R}+\sqrt{T}}{\sqrt{2}}\,|d\rangle + i\frac{\sqrt{R}-\sqrt{T}}{\sqrt{2}}\,|c\rangle\,; |\psi_5\rangle = \sqrt{T}\,|e\rangle + i\sqrt{R}\,|f\rangle$. Where we define T as the transmission coefficient and R as the reflection coefficient, such that $R+T = 1$. The two previous cases are recovered, respectively, for $T = 0,5$ and $T = 1,0$. When applying the BA criterion to corpuscular and wave phenomena, we will use Pauli matrices as their representative observables [8]. We define the σ_z operator as the observable that carries the system's path information, given that its eigenvectors refer to specific paths that do not overlap with each other. That said, since wave behavior is incompatible with corpuscular behavior, then its observable must be incompatible with σ_z. As a result, the observable that plays this role will be a Pauli matrix perpendicular to the z axis, which in this case will be a σ_y matrix.



For intermediate values of T we can construct the following graph for the respective realisms in $|\psi_1\rangle$.



Therefore, this treatment demonstrates that the Bohr's complementarity principle is not violated, since the sum of the realism of both behaviors is never greater than the maximum value of the realism of a single behavior.

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