Atomic movement in a partially cooled cavity

Thiago T. Tsutsui, Danilo Cius, Antonio Vidiella-Barranco, Antonio S. M. de Castro and Fabiano M. Andrade

***Abstract*— We investigate the resonant time-dependent Jaynes-Cummings model, where the atom crosses a partially cooled cavity. We find that, despite the field's thermal fluctuations, the dynamics of the atomic inversion and the Bloch vector become periodic due to the oscillatory atom-field coupling induced by the atomic motion.**

***Keywords*— Jaynes-Cummings model, Time-dependent coupling, Thermal state, Semiclassical atomic movement.**

I. Introduction

The Jaynes-Cummings (JC) model [1] remains a cornerstone in understanding quantum light-matter interaction, capturing the dynamics between an atom and a quantized electromagnetic field mode. In terms of applications, this model is employed in quantum information processing and quantum computation [3]. The time-dependent JC (TDJC) model [2] generalizes the model by allowing typically constant parameters to vary over time. The semiclassical atomic motion [2] may be accounted for with this extension. In this work, we investigate the resonant TDJC model. The scenario involves semiclassical atomic motion, captured by a time-dependent sinusoidal coupling parameter [4], within a partially cooled cavity initially prepared in a thermal state [5]. We focus on the behavior of the atom, analyzing the atomic probabilities and the motion of the Bloch vector [7]. We find that all the measured quantities have a behavior dominated by the periodicity of the sinusoidal coupling.

II. Time-dependent Jaynes-Cummings model

The interaction Hamiltonian for the TDJC model, considering atom-field resonance, is written as

$$\hat{V}(t) = \lambda(t)(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger). \tag{1}$$

The atomic degree of freedom is represented by the Pauli ladder operators ($\hat{\sigma}_\pm$), while the bosonic operators ($\hat{a}$ and $\hat{a}^\dagger$) are associated with the quantized cavity mode. The time-dependent coupling parameter is symbolized by $\lambda(t)$. The atomic levels are the excited ($|e\rangle$) and ground ($|g\rangle$) states, whereas the number state basis ($|n\rangle$) represents the excitations of the cavity mode. The time evolution operator reads

$$\begin{aligned}\hat{U}(t) = &\cos\left[A(t)\sqrt{\hat{a}^\dagger\hat{a}+1}\right]|e\rangle\langle e| + \cos\left[A(t)\sqrt{\hat{a}^\dagger}\right]|g\rangle\langle g| \\ &- i\hat{a}\frac{\sin\left[A(t)\sqrt{\hat{a}^\dagger\hat{a}+1}\right]}{\sqrt{\hat{a}^\dagger\hat{a}+1}}|e\rangle\langle g| \\ &- i\hat{a}^\dagger\frac{\sin\left[A(t)\sqrt{\hat{a}^\dagger\hat{a}+1}\right]}{\sqrt{\hat{a}^\dagger\hat{a}+1}}|g\rangle\langle e|,\end{aligned} \tag{2}$$

where $A(t) = \int_0^t \lambda(t')dt'$, corresponds to the coupling area.

III. Atomic movement in a partially cooled cavity

We analyze the time evolution of the population inversion, focusing on how the periodic nature of a sinusoidal coupling and a pronounced uncertainty of the thermal state influence the dynamics. The coupling parameter is modeled as

$$\lambda(t) = \lambda_0 \sin\left(\frac{p\pi v t}{L}\right), \tag{3}$$

where p represents the number of half-wave lengths of the field mode, v is the atomic velocity and L is the cavity length. From this point on, we assume that the velocity is $v = \zeta L/\pi$ and the field is initially in a thermal state,

$$\hat{\rho}_F(0) = \sum_{n=0}^{\infty} P_n |n\rangle\langle n|, \tag{4}$$

where

$$P_n = \frac{\langle n\rangle^n}{(1+\langle n\rangle)^{n+1}}, \tag{5}$$

is the Bose-Einstein distribution. The joint atom-field state evolves according to

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^\dagger(t). \tag{6}$$

where $\hat{U}(t)$ is the time evolution operator given in Eq. (2), and $\hat{\rho}(0)$ is the initial atom-field state.

For an initially excited atomic state, we analyze the atomic population inversion, $W(t) = \langle\hat{\sigma}_z\rangle = P_e(t) - P_g(t)$, where $P_e(t)$ and $P_g(t)$ represent the probabilities of the atom occupying the excited and ground states, respectively. The behavior of this observable is plotted in Fig. 1. The periodicity of the coupling dominates the time evolution of the population inversion.

The Bloch vector encodes rich information about a quantum state, capturing aspects such as its purity and the degree of atom-field alignment. The Bloch vector is defined as

$$\boldsymbol{R}(t) = (R_x(t), R_y(t), R_z(t)). \tag{7}$$

Thiago T. Tsutsui, Programa de Pós-Graduação em Ciências/Física, Universidade Estadual de Ponta Grossa, Ponta Grossa-PR, e-mail: takajitsutsui@gmail.com; Danilo Cius, Instituto de Física, Universidade de São Paulo, São Paulo-SP, e-mail: danilocius@gmail.com; Antonio Vidiella-Barranco, Instituto de Física "Gleb Wataghin", Universidade de Campinas, Campinas-SP, e-mail: vidiella@ifi.unicamp.br; Antonio S. M. de Castro, Departamento de Física e Programa de Pós-Graduação em Ciências/Física, Universidade Estadual de Ponta Grossa, Ponta Grossa-PR, e-mail: asmcastro@uepg.br; Fabiano M. Andrade, Departamento de Matemática e Estatística e Programa de Pós-Graduação em Ciências/Física, Universidade Estadual de Ponta Grossa, Ponta Grossa-PR e Departamento de Física, Universidade Federal do Paraná, Curitiba-PR, e-mail: fmandrade@uepg.br. This work was partially financed by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES, Finance Code 001). It was also supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Instituto Nacional de Ciência e Tecnologia de Informação Quântica (INCT-IQ). F.M.A. acknowledges CNPq Grant No. 313124/2023-0.

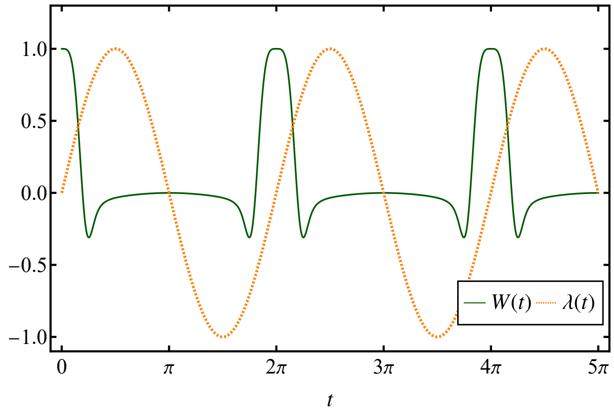

Fig. 1. The population inversion (solid green line) and the coupling parameter (dotted orange line) as a function of dimensionless time, when considering an average photon number of $\langle n\rangle = 25$, $\lambda_0 = \zeta = 1 = p = 1$.

Since any 2×2 matrix can be expressed as a linear combination of the Pauli matrices and the identity matrix $\hat{\mathbf{1}}_2$, the reduced atomic density matrix can be parametrized as

$$\hat{\rho}_A(t) = \frac{1}{2}[\hat{\mathbf{1}}_2 + \hat{\boldsymbol{\sigma}} \cdot \boldsymbol{R}(t)]. \tag{8}$$

To study the motion of the Block vector, we assume that the initial atomic state is given by an eigenvector of $\hat{\sigma}_x$, namely $|\psi_A(0)\rangle = 1/\sqrt{2}(|e\rangle + |g\rangle)$. In the density operator formalism, the initial state of the system reads

$$\hat{\rho}(0) = \left[\frac{1}{2}\left(|e\rangle\langle e| + |e\rangle\langle g| + |g\rangle\langle g| + |g\rangle\langle e|\right)\right] \otimes \hat{\rho}_F(0). \tag{9}$$

From the parametrization, Eq. (8), and from the density matrix, Eq. (6), we can calculate the components of the Bloch vector for the given initial state:

$$\begin{aligned} R_x(t) &= \sum_{n=0}^{\infty} P_n \cos\left[A(t)\sqrt{n}\right] \cos\left[A(t)\sqrt{n+1}\right], \\ R_y(t) &= 0, \\ R_z(t) &= \sum_{n=0}^{\infty} P_n \left\{\cos^2\left[A(t)\sqrt{n+1}\right] + \sin^2\left[A(t)\sqrt{n}\right]\right\} - 1. \end{aligned} \tag{10}$$

In Fig. 2, we present Bloch vector's motion for both the constant [Fig. 2(a)], and sinusoidal [Fig. 2(b)] coupling scenarios. For constant coupling, the Bloch vector shows a typical disordered motion [7], while the sinusoidal coupling leads to periodic motion.

IV. Conclusions

In this paper, we analyzed the TDJC model under resonance. We examined a scenario in which an atom moves inside a cavity that is not completely cooled, analyzing the resulting behavior of the population inversion and the Bloch vector. In contrast to the constant coupling case, we found that both the Rabi oscillations and the motion of the Bloch vector become periodic, governed by the sinusoidal coupling, even in the presence of the inherent fluctuations of the thermal field.

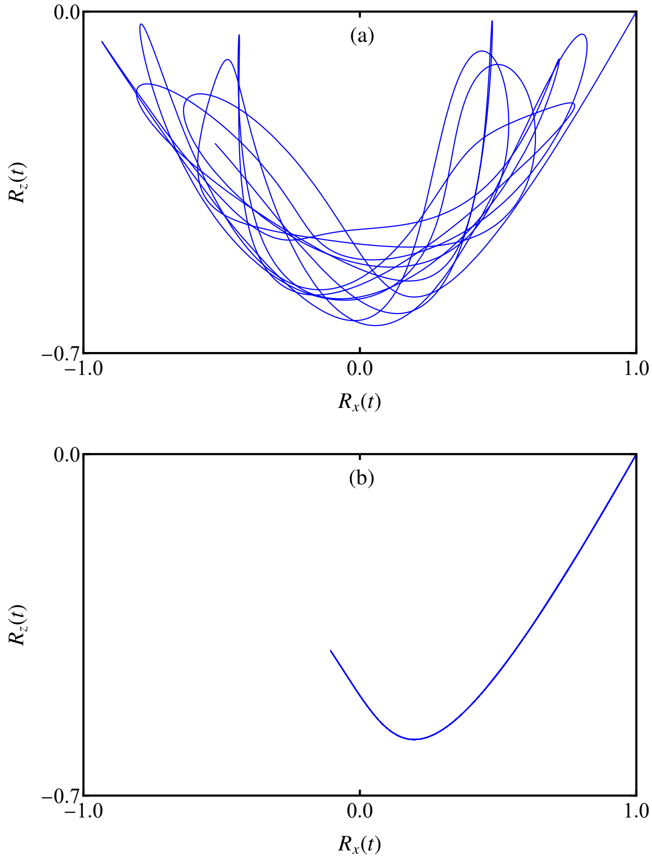

Fig. 2. Parameter plot of the $R_x(t)$ and $R_z(t)$ components, in regard to time. In panels (a), atomic motion is not considered. Conversely, in (b), atomic motion is included, with parameters $\lambda_0 = \zeta = p = 1$. In all cases, the initial average photon number is $\langle n\rangle = 0.5$.

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