

EFFECT OF POLYMERIC MATERIALS ON THE BAND STRUCTURE OF MECHANICAL METAMATERIALS

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ABSTRACT

Mechanical metamaterials (MM) are engineered structures used to control mechanical wave propagation, particularly through the formation of band gaps that block vibrations in specific frequency ranges. Despite extensive research, the combined effects of different materials and embedded resonators in MM thin plates remain few explored. In this context, this study investigates the formation of flexural band gaps associated with local resonance through the use of embedded resonators, focusing on the influence of different configurations of polymeric materials. Initially, a periodic unit cell with a square lattice was modeled using shell elements within the finite element method (FEM), with simulations performed in COMSOL Multiphysics. Periodic boundary conditions were applied via Bloch's theorem. The configuration with Verowhite in the plate and polyvinyl chloride (PVC) in the resonator showed the best performance, achieving the largest total bandwidth ($BW_{TOT} = 372.10$ Hz) while maintaining intermediate mid frequencies. Compared to other configurations, it provided bandwidth gains of 67.4% over the fully Verowhite model, 15.2% over the fully PVC structure, and 83.6% over the PVC/Verowhite setup. The results confirm that the use of resonators with higher stiffness promotes the enlargement of band gaps and that the appropriate combination of materials enables greater control over the attenuation range and the positioning of the gaps, outperforming single-material configurations.

Keywords: Thin plates, flexural vibration control, complete band gaps, embedded resonators, finite elements.

INTRODUCTION

Metamaterials are artificial materials engineered to exhibit physical properties not found in nature [1,2]. Designed to meet specific mechanical requirements, mechanical metamaterials (MM) allow the tuning of properties such as stiffness, strength, and energy absorption. MM are applied in various engineering fields, including civil, aerospace, and automotive sectors [3,4], and are highly effective in dissipating mechanical energy under high-impact or continuous vibration conditions. Additionally, they can block or redirect vibrations, making them ideal for

isolating sensitive equipment and systems [5]. Vibration control in MM is achieved through frequency ranges in which mechanical waves cannot propagate, known as band gaps, providing an effective solution for vibroacoustic isolation in multiple applications [6].

Band gaps can be tuned through modifications in the structure's geometry or in the properties of the materials employed. This phenomenon may occur via Bragg scattering in periodic structures, where the destructive interference of waves interacting with the structure's periodicity generates a frequency range corresponding to the band gaps. In this case, the frequencies depend on the wavelength and the size of the unit cell (i.e., the smallest repetitive element of a periodic structure) [7]. On the other hand, band gaps can also be generated through the mechanism of local resonance, due to the presence of resonant elements within the structure. This phenomenon can lead to band gaps at lower frequencies, which are directly related to the properties of the local resonators, regardless of the unit cell size [8].

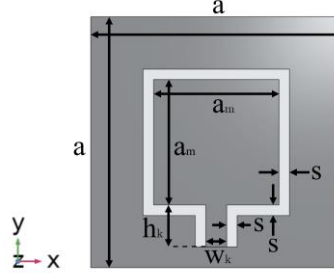
The formation of flexural band gaps in plate structures has been explored by the scientific community as an innovative approach for vibration control [9–12]. However, there remains a gap in the literature regarding the simultaneous consideration of multiple factors, especially when dealing with MM thin plate structures with embedded resonators. Band structure investigations in plates by the finite element method (FEM) have focused on both two-dimensional [13,14] and three-dimensional elements [15], employing strategies that incorporate the phenomenon of local resonance. Among these, 2D elements stand out due to their lower computational cost compared to 3D models, making them ideal for parametric analyses involving a wide range of parameters. Shell elements simplify the 3D model to a 2D surface while maintaining accuracy, making them well-suited for structures whose thickness is much smaller than their length and width [16].

In this context, this study investigates the formation of flexural band gaps associated with local resonance in 2D MM thin plates incorporating embedded resonators. The analysis focuses on the effects of different polymeric material configurations on the resulting band structure. To achieve this, numerical simulations are performed using FEM with shell elements, allowing for an accurate representation of the system's dynamic behavior. A two-dimensional periodicity in a square lattice is considered through Bloch's theorem. This paper is organized as follows: Section 2 presents the modeling of the 2D MM thin plate; Section 3 provides the numerical results for the polymeric materials Verowhite and polyvinyl chloride (PVC), investigating four main configurations: (i) single-material Verowhite in both the plate and the resonator, (ii) single-material PVC in both the plate and the resonator, (iii) Verowhite in the plate and PVC in the resonator, and (iv) PVC in the plate and Verowhite in the resonator. Finally, Section 4 presents the concluding remarks.

MODELING OF THE 2D MM THIN PLATE

In this work, a 2D thin plate associated with the concept of MM with embedded resonators was considered. All numerical results were obtained through the FEM, using COMSOL Multiphysics software with shell elements. The structure was composed of an isotropic polymeric material commonly used in 3D printing, known as Verowhite. The mechanical properties adopted for this material were: Young's modulus (E) of 1.621 GPa, density (ρ) of 1187.10 kg/m³, and Poisson's ratio (ν) of 0.308 [14]. Figure 1 shows the schematic of the unit cell of a square lattice composed of a resonator (m) and a spring (k). The geometrical parameters of the structure are represented by the lattice parameter ($a = 12$ mm), resonator width ($a_m = 6$ mm), spring height ($h_k = 2$ mm), spring width ($w_k = 1$ mm), and vacuum cavity ($s = 0.50$ mm).

The flexural vibration along the z -axis was investigated through the analysis of infinite MM thin plates (in the xy -plane). For this purpose, a shell element with thickness (t_h) equal to 1 mm was used, with displacements constrained to zero in the x and y directions. Accordingly, the



band structures were obtained by considering two-dimensional periodicity, where the Figure 1 – Dimensions of the unit cell of the 2D MM thin plate composed of Verowhite polymeric material (plate and resonator), showing the lattice parameter (a), resonator width (a_m), spring height (h_k), spring width (w_k), and vacuum cavity (s).

displacement vector was described using Bloch's theorem as:

$$\mathbf{u}(\mathbf{r}, t) = e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \mathbf{u}_{\mathbf{k}}(\mathbf{r}) \quad (1)$$

In this case, $\mathbf{r}$ is the position vector, t represents time, ω is the angular frequency, and $\mathbf{u} \equiv (u, v, w)$, with $u(x, y, z)$, $v(x, y, z)$, and $w(x, y, z)$ representing the displacements in the x , y , and z directions, respectively. In the infinite configuration, adjacent unit cells are spaced by a in the xy -plane, following a square lattice. In this context, the Bloch wave vector follows the 2D periodicity (in a square lattice) and is expressed in terms of its components as $\mathbf{k} = (k_x, k_y)$.

The band structure $\omega(\mathbf{k})$, i.e., the dispersion relation of the propagating waves, where ω is the angular frequency and $\mathbf{k}$ is the wave vector, was analyzed within the first irreducible Brillouin zone (FIBZ), as shown in Fig. 2. The displacement was predescribed with both x and y components set to zero, focusing solely on the transverse, i.e., flexural displacement. After performing the modal analysis of the unit cell and generating the band structure along with the corresponding wave mode shapes, a new material was introduced into the structure, considering different combinations. For this purpose, PVC was selected, whose mechanical properties adopted in this study were: density (ρ) of 1204.80 kg/m³, Young's modulus (E) of 3.50 GPa, and Poisson's ratio (ν) of 0.319 [17]. For the simulations, a refined mesh consisting of a total of 1872 triangular surface elements was used.

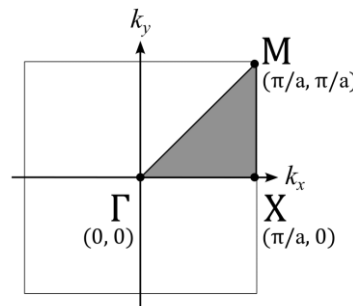


Figure 2 – FIBZ highlighted by the triangle for a square lattice bounded by Γ , X , and M .

RESULTS AND DISCUSSION

To investigate the influence of polymeric materials on the dynamic properties of a 2D MM thin plate, the material Verowhite was initially employed. Floquet–Bloch periodic boundary conditions were applied, generating the band structure and the corresponding wave mode shapes, as shown in Fig. 3. The formation of partial band gaps (PBG) was observed along the paths $\Gamma \rightarrow X$, $X \rightarrow M$, and $M \rightarrow \Gamma$ within various frequency ranges (highlighted by light gray

rectangles), as well as the appearance of two complete band gaps (CBG), clearly indicated in Fig. 3a (highlighted by dark gray rectangles). The first complete band gap (CBG¹) was characterized by the wave mode shapes corresponding to the lower limit ($V_1 = 952.38$ Hz) and the upper limit ($V_2 = 1131.40$ Hz), resulting in a bandwidth (BW) of 179.02 Hz. For the second complete band gap (CBG²), the lower ($V_3 = 2374.80$ Hz) and upper ($V_4 = 2418.10$ Hz) bounds defined a BW of 43.30 Hz.

Figure 3b revealed the opening of CBG¹, associated with the bending modes of the resonator, while CBG² was related to the torsional modes of the resonator, both preventing wave propagation along the structure of the thin plate. The mid frequencies (MF, i.e., the arithmetic mean between the lower and upper limits of the CBG) of CBG¹ and CBG² were 1041.89 Hz and 2396.45 Hz, respectively, and the sum of the BW of these regions resulted in a total bandwidth (BW_{TOT}, i.e., the BW sum of CBG¹ and CBG²) of 222.32 Hz. Finally, the wave mode shapes V_5 and V_6 exhibited significant plate displacement concomitant with the resonator's action, giving rise to a narrow PBG along the $\Gamma \rightarrow X$ path, between 3114.10 Hz and 3134.80 Hz, with BW = 20.70 Hz. Based on the analysis of the band structure of the initial model, the investigation begins on how different material configurations can influence the formation of CBG¹ and CBG², with emphasis on the 0 to 5000 Hz range. The following simulations analyze in more detail how the properties of Verowhite and PVC, acting individually or in combination, impact the band structure, with a focus on BW and MF.

Figure 4 presents the band structures obtained for different material combinations in the plate/resonator: single material in Verowhite/Verowhite (Fig. 4a), single material in PVC/PVC (Fig. 4b), combined material with PVC/Verowhite (Fig. 4c), and combined material with Verowhite/PVC (Fig. 4d). The main results are shown in Table 1, where the MF and BW of CBG¹ and CBG², as well as the BW_{TOT}, are indicated. The higher stiffness of PVC resulted in a shift of the MF of CBG¹ and CBG² to higher frequencies, in addition to promoting a widening of the BW of both CBG when compared to Verowhite. The band structure for the single PVC material (Fig. 4b) showed a BW_{TOT} of 323.10 Hz, corresponding to an increase of 45.33% compared to the single Verowhite material (Fig. 4a), which had 222.32 Hz. Furthermore, increases of 45.69% in the MF of CBG¹ (from 1041.89 to 1517.90 Hz) and 45.38% in the MF of CBG² (from 2396.45 to 3484.05 Hz) were observed.

On the other hand, the combination of PVC in the plate and Verowhite in the resonator (Fig. 4c) resulted in a BW_{TOT} of 202.70 Hz, a value lower than that of both the single Verowhite

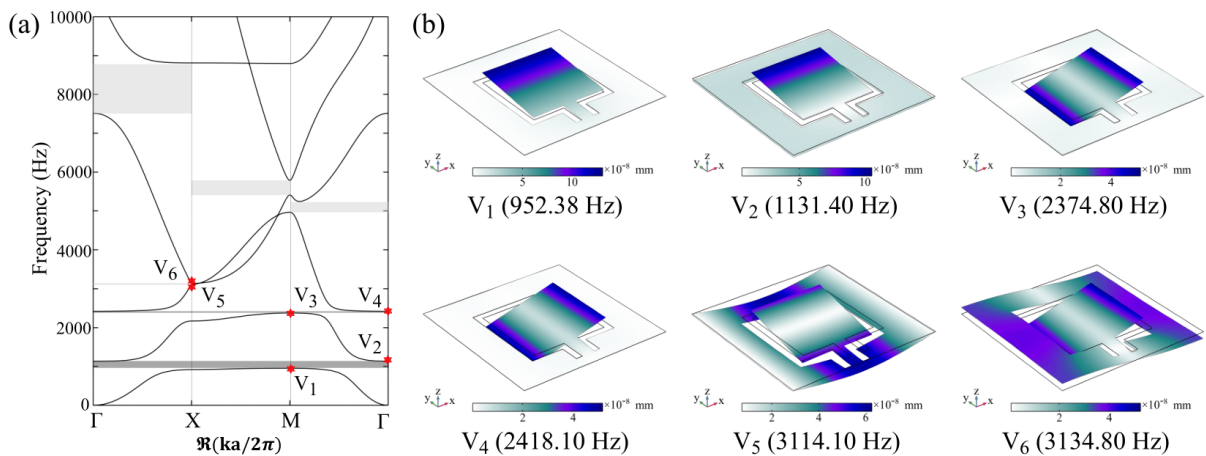


Figure 3 – (a) Band structure for a MM thin plate composed of Verowhite in the frequency range from 0 to 10000 Hz and (b) the corresponding wave mode shapes ($V_1 - V_6$).

material (222.32 Hz) and the single PVC material (323.10 Hz), indicating that the combination was not effective in widening the total attenuation range. The reduction in the BW of CBG² (19.30 Hz) compared to the other configurations was the main factor for this decrease, along with the shift of the MF of CBG¹ to higher frequencies (1192.60 Hz compared to 1041.89 Hz in the single Verowhite material), which may be undesirable depending on the target attenuation range. When compared to the single PVC material (Fig. 4b), the PVC/Verowhite configuration (Fig. 4c) showed a 37.29% drop in BW_{TOT} (from 323.10 to 202.70 Hz), highlighting that replacing the resonator with Verowhite compromised the total bandwidth. Relative to the single Verowhite material (Fig. 4a), an 8.81% reduction was observed (from 222.32 to 202.70 Hz), indicating inferior performance even with the introduction of PVC in the plate. These results suggest that, although PVC improved the MF of CBG¹, the use of Verowhite in the resonator limited the structure's ability to form wide CBG.

When comparing the different material arrangements used in the unit cell, it is observed that the combination with a Verowhite plate and a PVC resonator (Fig. 4d) showed the best performance in terms of BW_{TOT}. This arrangement resulted in a BW_{TOT} of 372.10 Hz, representing a 67.4% increase (from 222.32 Hz to 372.10 Hz) compared to the single Verowhite material configuration (Fig. 4a). In comparison with the single PVC material arrangement (Fig. 4b), the BW_{TOT} increased by 15.2% (from 323.10 Hz to 372.10 Hz). Relative to the inverse combination with PVC in the plate and Verowhite in the resonator (Fig. 4c), the improvement was even more significant, reaching 83.6% (from 202.70 Hz to 372.10 Hz).

Analyzing separately the contributions of CBG¹ and CBG², it is verified that the BW of CBG¹ for the combined configuration with Verowhite in the plate and PVC in the resonator

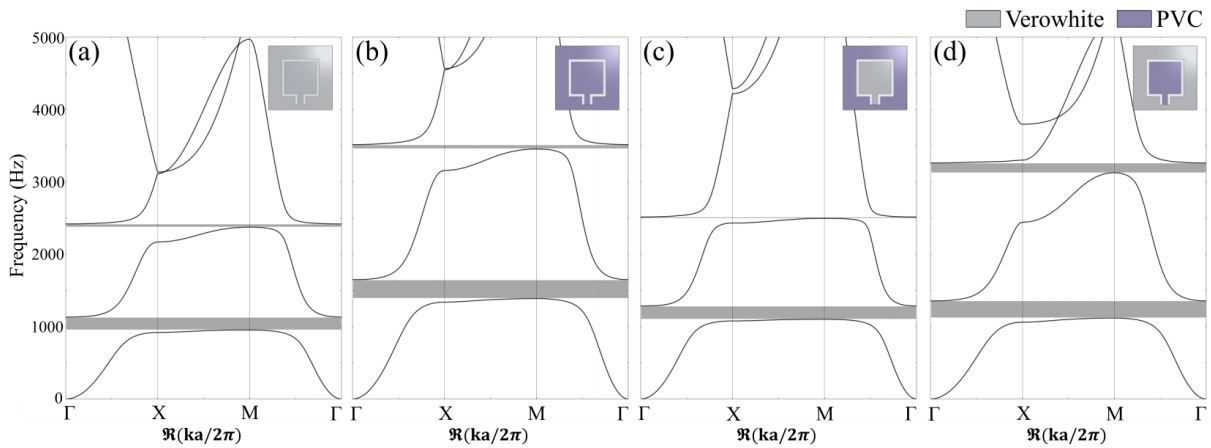


Figure 4 – Band structure for the unit cell ($t_h = 1$ mm, $a = 12$ mm) considering the material of the plate and resonator (plate/resonator): (a) single material with Verowhite/Verowhite; (b) single material with PVC/PVC; (c) combined material with PVC/Verowhite; and (d) combined material with Verowhite/PVC.

Table 1 – Numerical results according to the unit cell material, showing the MF and BW corresponding to CBG¹ and CBG², as well as the total bandwidth (BW_{TOT} = BW CBG¹ + BW CBG²).

Plate Material	Resonator Material	MF CBG ¹ (Hz)	BW CBG ¹ (Hz)	MF CBG ² (Hz)	BW CBG ² (Hz)	BW _{TOT} (Hz)
Verowhite	Verowhite	1041.89	179.02	2396.45	43.30	222.32
PVC	PVC	1517.90	261.40	3484.05	61.70	323.10
PVC	Verowhite	1192.60	183.40	2505.15	19.30	202.70
Verowhite	PVC	1236.45	236.70	3193.30	135.40	372.10

(Fig. 4d) was 32.3% higher than that of the single Verowhite material system (from 179.02 Hz to 236.70 Hz) and 29.1% higher than the configuration with PVC in the plate and Verowhite in the resonator (Fig. 4c), which had 183.40 Hz. However, compared to the single PVC material system (Fig. 4b), a slight reduction of 9.4% was observed (from 261.40 Hz to 236.70 Hz) in the BW of CBG¹. On the other hand, the BW of CBG² showed significant increases in the arrangement with Verowhite in the plate and PVC in the resonator. Compared to the arrangements with Verowhite in both the plate and resonator, with PVC in both components, and with PVC in the plate and Verowhite in the resonator, the increases were 212.8% (from 43.30 Hz to 135.40 Hz), 119.4% (from 61.70 Hz to 135.40 Hz), and 601.0% (from 19.30 Hz to 135.40 Hz), respectively. These results indicate that the combination of Verowhite in the plate with PVC in the resonator significantly favors the widening of the CBG², which is the main factor responsible for the increase in the BW_{TOT}.

Moreover, it is observed that the average frequencies associated with the CBG¹ and CBG² bands for the configuration with Verowhite on the plate and PVC in the resonator, 1236.45 Hz and 3193.30 Hz respectively, lie between the values obtained for systems composed exclusively of Verowhite (1041.89 Hz and 2396.45 Hz) and exclusively of PVC (1517.90 Hz and 3484.05 Hz). This intermediate behavior in the positioning of the CBG can be advantageous for applications that require selective control of specific frequencies. In this context, the proper choice of material combinations can be an effective strategy to maximize the BW of CBG in MM thin plates without shifting the MF to higher frequencies. Furthermore, it was found that the use of the stiffer material in the resonator significantly contributes to the enhancement of the system's dynamic properties, favoring the broadening and positioning of the CBG. It is possible that, when done carefully, the combination of materials is more efficient than using a single material throughout the structure, which may allow greater flexibility in adjusting the desired dynamic properties.

CONCLUSIONS

This study evaluated the influence of polymeric materials on the dynamic behavior of band structures in mechanical metamaterial thin plates, modeled as 2D shell elements via the finite element method. The embedded resonators within the thin plate structure were capable of generating both complete and partial flexural band gaps in the band structure. The initial analysis identified the formation of two complete band gaps, associated with locally resonant bending and torsional modes of the embedded resonator. It was observed that the material configuration (Verowhite or PVC) significantly affects the opening, position, and width of the complete band gaps. The study demonstrated that stiffer materials tend to produce wider band gaps, though they shift the mean frequencies to higher values. On the other hand, appropriate combinations of materials for the plate and resonator can enhance the dynamic performance compared to using a single material throughout the structure.

The configuration with Verowhite in the plate and PVC in the resonator showed the best performance among all combinations, yielding the highest total bandwidth of 372.10 Hz. This represented an improvement of 67.4% compared to the Verowhite-only model, 15.2% compared to the PVC-only model, and 83.6% compared to the inverse combination (PVC in the plate and Verowhite in the resonator). The key feature of this configuration was the significant enlargement of the second band gap, which reached a width of 135.40 Hz, far superior to the others. For future work, a parametric analysis involving alternative materials and geometric variations is recommended, as well as an investigation into the influence of

mechanical loss associated with the voided section and material selection. Additionally, the application of genetic algorithms is suggested to optimize the dynamic properties and enhance the practical application potential of such structures.

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